Adaptive Algorithms for the Rejection of Sinusoidal Disturbances with Unknown Frequency

Marc Bodson and Scott C. Douglas *
Department of Electrical Engineering
University of Utah
Salt Lake City, UT 84112
(801) 581 8590 and *(801) 581 4445*
bodson@ee.utah.edu and *douglas@ee.utah.edu*
Fax: (801) 581 5281

Abstract

Two algorithms are presented for the rejection of sinusoidal disturbances with unknown frequency. The first is an indirect algorithm where the frequency of the disturbance is estimated, and the estimate is used in another adaptive algorithm that adjusts the magnitude and phase of the input needed to cancel the effect of the disturbance. A direct algorithm that uses the concept of a phase-locked loop is also presented in which frequency estimation and disturbance cancellation are performed simultaneously. Approximate analyses are presented for both schemes and the results are found useful for the selection of the design parameters. Simulations are given which demonstrate the validity of the analytical results and the ability of the algorithms to reject sinusoidal disturbances with unknown frequency. The indirect algorithm is found to have a larger capture region for the parameter estimates, whereas the direct algorithm has superior convergence properties locally about the optimum parameter estimates.

Keywords: adaptive control, adaptive signal processing, sinusoidal disturbances, periodic disturbances, phase-locked loops.

*This material is based upon work supported by the U.S. Army Research Office under grant number DAAH04-96-1-0085. The content of the paper does not necessarily reflect the position or the policy of the federal government, and no official endorsement should be inferred.

1 Introduction

In this paper, we consider the problem of attenuating an output signal $y(t)$ that, in the Laplace domain, is given by

$$y(s) = P(s)(u(s) - d(s)), \tag{1}$$

where $u(s)$ and $d(s)$ are the Laplace transforms of the controller output and disturbance signals, respectively, and $P(s)$ is the Laplace transform of the impulse response of the plant. The plant is assumed to be linear, time-invariant, and stable. The goal of the control system is to generate $u(t)$ such that $y(t) \to 0$ as $t \to \infty$. We assume that the disturbance $d(t)$ is a sinusoid of fixed magnitude ${\theta_1}^*$ and fixed frequency ω_1 such that

$$\begin{aligned} d(t) &= {\theta_1}^* \cos(\alpha_d(t)), \\ \dot{\alpha}_d(t) &= \omega_1. \end{aligned} \tag{2}$$

In this task, the parameters ${\theta_1}^*$, ω_1, and $\alpha_d(0)$ are all unknown.

Algorithms for solving the above problem have use in a wide range of applications. Of particular interest is the problem of active noise and vibration control, in which $d(t)$ is an offending noise or vibration source and $P(s)$ is the transfer function of the output-actuator-to-error-sensor propagation path. Often, the noise source consists mainly of periodic components due to rotating machinery generating the undesired noise signal. Examples of such noises include engine noise in turboprop aircraft (Emborg and Ross, 1993), engine noise in automobiles (Shoureshi and Knurek, 1996), and ventilation noise in HVAC systems (Eriksson, 1988). In practice, the frequency of the disturbance is usually not known and may even vary during operation. In these cases, it is desirable to place an encoder or tachometer on the rotating machine that is at the origin of the disturbance to measure the frequency of the disturbance. Alternatively, if a sensor can be placed near the source of the disturbance such that $d(t)$ can be accurately characterized, then the control task reduces to that of adaptive feedforward control (AFC) (see Nelson and Elliott, 1992). However, it is impossible to use such a sensor in applications where the addition of the sensor is too costly or reduces the reliability of the overall system due to strong vibrations, high temperatures, or dirty conditions within the environment.

In this paper, we present two adaptive control algorithms for the rejection of sinusoidal disturbances with unknown frequency. These systems only require a single sensor located at the output of the plant for their operation. The first approach combines an AFC scheme for attenuating sinusoidal disturbances with known frequency as described and analyzed in Bodson *et. al.* (1994) and Sacks *et. al.* (1996), together with an algorithm to estimate the frequency of the disturbance. This approach is called *indirect* because the frequency of the disturbance is estimated independently of the cancellation scheme. The second approach consists in extending the AFC scheme of Bodson *et. al.* (1994) and Sacks *et. al.* (1996) by integrating a *phase-locked loop* within the scheme so that disturbances with unknown frequency can be directly cancelled. Our analyses of these two methods enable the design of the systems to provide useful rejection of sinusoidal disturbances, and simulations validate the theoretical results. Our results indicate that the direct algorithm has superior convergence properties locally about the optimum convergent point of the controller, whereas the indirect algorithm provides a wider capture region for the system's adaptive parameters.

2 Indirect Approach

2.1 Frequency Estimation

Our development of the indirect algorithm relies on a method for the estimation of the frequency of a signal. Here, we consider an adaptive notch filter developed in Regalia (1991) that is designed to eliminate one (or more) periodic component(s) from a measured signal. This adaptive notch filter estimates the frequency of the unknown signal as part of its operation. In our work, we transpose Regalia's algorithm to continuous-time so that standard averaging methods can be used to analyze the system's equations. It is relatively straightforward to transfer the results of our control system design back to the discrete-time domain for implementation purposes.

Consider the problem of estimating the frequency ω of a signal $y(t)$, where θ_f is the resulting estimate. The continuous-time version of Regalia's algorithm has three states θ_f, x_1, and x_2, that satisfy the differential equations

$$\dot{x}_1 = x_2, \tag{3}$$

$$\dot{x}_2 = -2\zeta\theta_f x_2 - {\theta_f}^2 x_1 + ky, \tag{4}$$

$$\dot{\theta}_f = -g_1(ky - 2\zeta\theta_f x_2)x_1. \tag{5}$$

Note that this system of equations is highly nonlinear. The three parameters of the system are the adaptation gain g_1, the damping factor ζ, and a filter gain k, all of which are positive-valued. The algorithm's behavior can be explained through an averaging analysis, justified for small values of g_1 and for a periodic signal $y\,(t)$ (*c.f.* Sastry and Bodson (1989)). For θ_f constant,

$$x_1 = \frac{k}{D(s,\theta_f)}\;[y], \qquad x_2 = \frac{ks}{D(s,\theta_f)}\;[y], \tag{6}$$

where $D(s,\theta_f) = s^2 + 2\zeta\theta_f s + {\theta_f}^2$. Furthermore,

$$ky - 2\zeta\theta_f x_2 = \frac{k(s^2 + {\theta_f}^2)}{D(s,\theta_f)}\;[y]. \tag{7}$$

By application of averaging theory, the evolution of the state θ_f can be approximated by the solution of the averaged system, which is given by

$$\dot{\theta}_{av} = -g_1\;AVG\;\left[\frac{k(s^2+\theta_{av}^2)}{D(s,\theta_{av})}\;[y]\;\;\cdot\;\frac{k}{D(s,\theta_{av})}\;[y]\right]. \tag{8}$$

Assuming that $y(t) = \sum_{i=1}^{n} r_i \sin(\omega_i t + \phi_i)$, the averaged system is given by

$$\dot{\theta}_{av} = -\frac{g_1}{2}\sum_{i=1}^{n}\frac{k^2 r_i^2\left(\theta_{av}^2 - \omega_i^2\right)}{\left(\theta_{av}^2 - \omega_i^2\right)^2 + \left(2\zeta\theta_{av}\omega_i\right)^2}. \tag{9}$$

If the signal $y(t)$ has a single sinusoidal component, the analysis indicates that the averaged system has a pair of equilibrium points at $\theta_{av} = \pm\omega_1$. Around ω_1, the linearized system is exponentially stable, with dynamics

$$\dot{\theta}_{av} = -\frac{g_1 k^2 {r_1}^2}{4\zeta^2{\omega_1}^3}(\theta_{av} - \omega_1) \tag{10}$$

In other words, the algorithm is able to identify the frequency of the signal ω_1. Even in the presence of multiple sinusoids, the algorithm is able to lock onto a main frequency component despite the presence of competing tones. Typically, the averaged system will exhibit multiple equilibrium points. If the parameter ζ is small, the first term in the sum of (9) will dominate the others when θ_{av} is close to ω_1. Therefore, in the neighborhood of $\theta_{av} = \omega_1$, the right-hand side of (9) will be approximately the same as that of the system with only one sinusoidal component. The same argument can be repeated for the other frequencies, so that there will be an equilibrium point associated with each of the sinusoidal components of the signal $y(t)$. The initial value of θ_f and the magnitudes of the frequency components r_i affect the frequency towards which θ_f will converge.

2.2 Indirect Adaptive Algorithm

The algorithm for frequency estimation can be combined with the adaptive feedforward control scheme in Sacks *et. al.* (1996) to produce an algorithm capable of attenuating periodic disturbances with unknown frequency. The control law is given by

$$\begin{aligned} u(t) &= \theta_c(t)\cos(\alpha(t)) - \theta_s(t)\sin(\alpha(t)), \\ \dot{\alpha}(t) &= \theta_f(t), \end{aligned} \tag{11}$$

where the two adaptive parameters θ_c and θ_s are updated as

$$\frac{d}{dt}\begin{bmatrix} \theta_c \\ \theta_s \end{bmatrix} = -g_2\, G^{-1} \begin{bmatrix} y\cos(\alpha) \\ -y\sin(\alpha) \end{bmatrix}, \tag{12}$$

and

$$G = \frac{1}{2}\begin{bmatrix} P_R & -P_I \\ P_I & P_R \end{bmatrix}; \quad P_R = Re[P(j\omega_1)]; \quad P_I = Im[P(j\omega_1)]. \tag{13}$$

The parameter $g_2 > 0$ is an arbitrary adaptation gain. The averaging analysis of (12)–(13) shows that, for small values of the gain g_2, the dynamics of this adaptive system are approximately the same as those of two decoupled, first-order systems, with their poles located at $-g_2$ (rad/s).

Note that the algorithm in (12)–(13) uses the value of the frequency response of the plant at the frequency of the disturbance ω_1. However, the stability of (12) for small gain g_2 is guaranteed so long as the phase of the frequency response is correct to within ± 90 degrees.

Therefore, a rough estimate of ω_1 in (12) will be adequate in most cases. If desired, ω_1 may also be replaced by θ_f in (13).

2.3 Simulations of the Indirect Algorithm

We now present simulations of the indirect algorithm for a plant given by $P(s) = 100/(s+100)$ and a sinusoidal disturbance $d(t) = \cos(100t)$. In the frequency estimation algorithm, we choose the initial states to be equal to zero, and we choose $\zeta = 0.1$, $k = 100$, and $g_1 = 1000$. With these parameter choices, the pole of the linearized system in (10) is at -125 rad/s. In the AFC scheme, the initial states of the system are set to zero, and we choose $g_2 = 10$. Such a parameter choice yields an averaged system with two real poles at -10 rad/s, as predicted by the analysis in Sacks *et. al.* (1996).

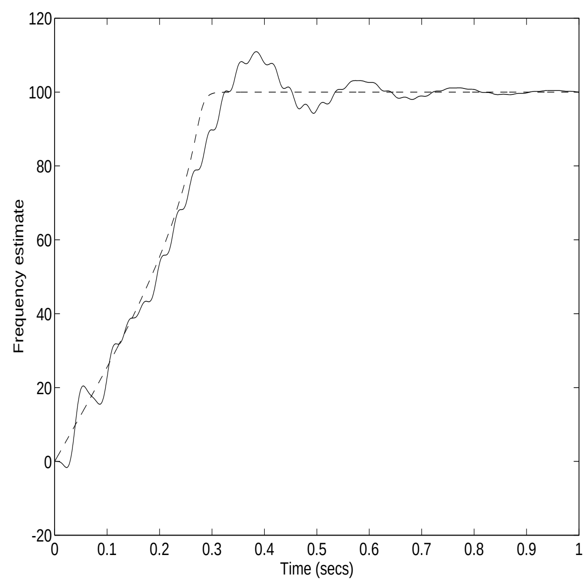

Figure 1: Frequency estimate for the indirect algorithm – Separate Adaptation

For our first simulation, frequency estimation and disturbance cancellation are performed separately. Specifically, for the first second of the simulation, the parameters of the AFC scheme are frozen while an accurate estimate of the frequency of the disturbance is calculated. Then, the frequency estimate is fixed for $t \geq 1$ and is used by the AFC scheme to adjust the amplitudes of the $\cos(\cdot)$ and $\sin(\cdot)$ components of the controller to attenuate the disturbance.

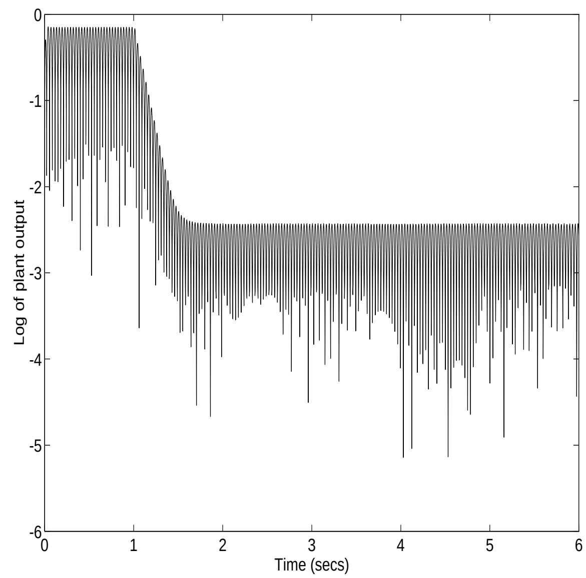

Figure 2: Log of the plant output for the indirect algorithm – Separate Adaptation

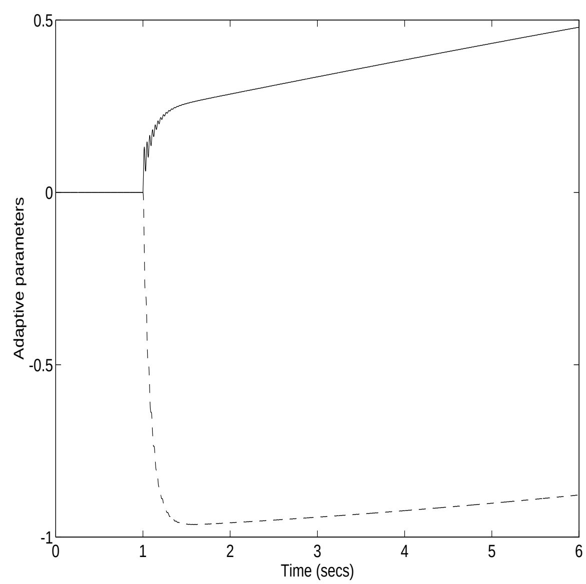

Figure 3: Adaptive parameters for the indirect algorithm – Separate Adaptation

Fig. 1 shows the frequency estimate produced by the algorithm, as indicated by the solid line in the figure. For our parameter choices, the frequency estimation algorithm in (12)–(13) converges within the first second of the simulation to the value $\theta_f = 99.95$ rad/s, a value close to $\omega_1 = 100$ rad/s. The response predicted by the nonlinear averaged system in (8) is shown as a dashed line on this figure and is seen to closely approximate the true system's transient response. Note that the frequency estimation algorithm converges to its proper setting despite a 100% initial error in the frequency estimate, and the ability of this algorithm to lock onto the frequency of the unknown disturbance is useful in many practical situations.

Fig. 2 shows the logarithm of the output of the plant, computed as $\log(|y(t)| + \epsilon)$ where $\epsilon = 10^{-6}$. The envelope of the plant output is constant during the frequency estimation phase of the controller's operation and is seen to decrease rapidly once the AFC algorithm is engaged. The system's output does not decrease significantly after a period of time. The evolutions of θ_c and θ_s are shown in Fig. 3 as a solid line and a dashed line, respectively. While the parameters appear to be drifting for $t > 2$ s, their variation is in fact sinusoidal at a frequency equal to the difference between the true and estimated frequencies of disturbance, and the sum of the squares of the parameters remains close to one over this time period. Note that continuous variations of the amplitudes of the $\cos(\cdot)$ and $\sin(\cdot)$ components of the controller output are necessary to reduce the disturbance amplitude when the frequency estimate θ_f is not exact, resulting in a low residual output error in steady-state.

We now explore the performance of the indirect scheme in a true adaptive mode in which all the parameters are adjusted simultaneously. Although this implementation might be expected to resolve the convergence problems observed with separate parameter adaptation stages, such is not the case. The log of the plant output is shown in Fig. 4. Although this implementation yields a more rapid decrease of the plant output, a substantial residual error remains at the end of the simulation run. In this case, the lack of asymptotic convergence may be attributed to the fact that the control input eliminates the signal which is used for the frequency estimation, thus preventing accurate convergence of θ_f. This problem is an

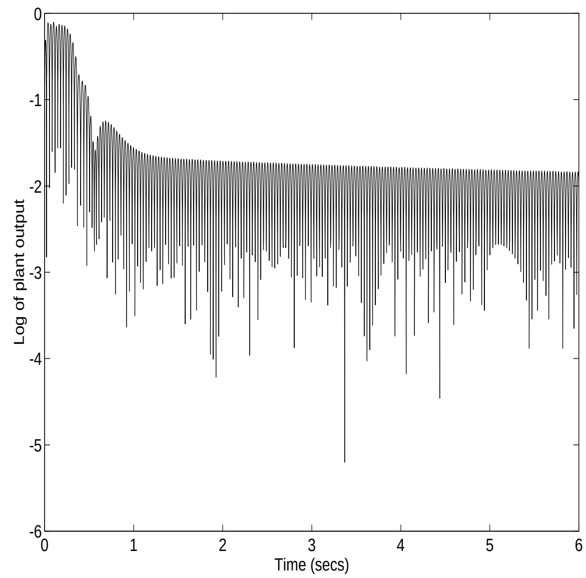

Figure 4: Log of the plant output for the indirect algorithm – Simultaneous adaptation

indication of the fact that the indirect scheme is overparameterized and can be resolved by considering a direct adaptive scheme in which the amplitude and phase of the sinusoidal component are computed via a single combined procedure, as we now show.

3 A Direct Algorithm

3.1 Adaptive Algorithm

An alternative approach to the indirect scheme of the previous section is a direct scheme in which a single error signal is used to update the frequency and the magnitude estimates simultaneously. Here, one such algorithm is presented that combines elements of the AFC scheme discussed earlier with a modified version of a phase-locked loop structure commonly used in communication systems (Hambley, 1990).

This scheme is shown in Fig. 5. In the figure, θ_1 is the estimate of the magnitude of the disturbance signal, and θ_2 is the estimate of its instantaneous frequency. Moreover, α is the estimate of the phase of the disturbance signal and is the integral of the estimate of the frequency θ_2. The equations for the control algorithm are

$$u = \theta_1 \cos(\alpha),$$

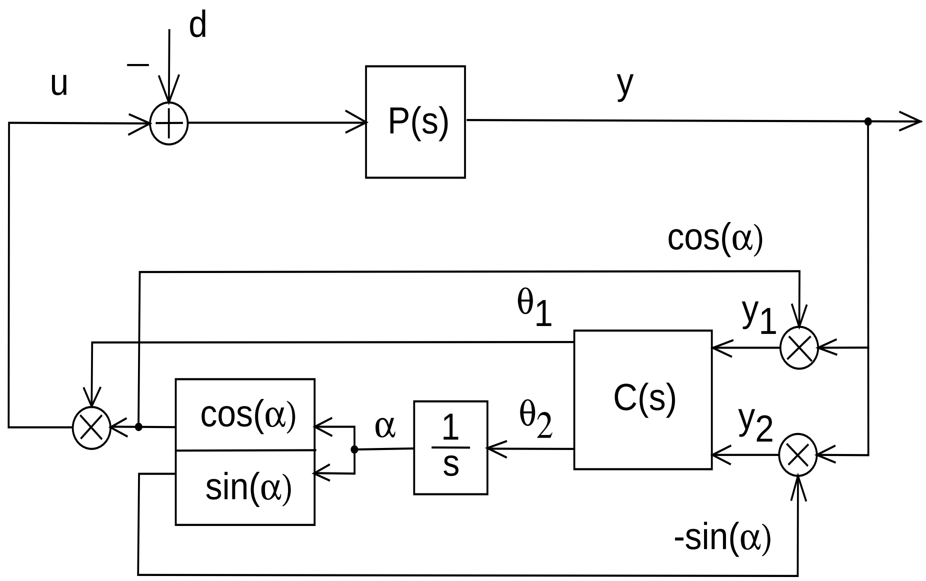

Figure 5: Direct Adaptive algorithm for sinusoidal disturbance cancellation

$$
\begin{aligned}
\dot{\alpha} &= \theta_2, \\
y_1 &= y\cos(\alpha), \\
y_2 &= -y\sin(\alpha).
\end{aligned} \tag{14}
$$

The user-defined transfer function matrix $C(s)$ relates the signals y_1 and y_2 to the parameters θ_1 and θ_2, respectively. An approximate analysis of this adaptive system will allow us to deduce a simple procedure for the design of $C(s)$ to obtain adequate performance from this system.

3.2 Approximate Analysis

Our analysis is based on a fundamental fact, whose proof is reminiscent of derivations found in the analysis of frequency-modulation communication systems (*c.f.* Hambley, 1990).

Assumptions:

- The values of θ_1 and θ_2 vary sufficiently slowly that the response of the plant to the signal $u(t)$ can be approximated by the steady-state output of the plant for a sinusoidal input with frequency θ_2.

- The instantaneous frequency θ_2 is close to ω_1, such that $P(j\theta_2)$ can be replaced by $P(j\omega_1)$.

Basic Fact: Considering low-frequency components only, the two signals $y_1(t)$ and $y_2(t)$ are approximately given by

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = G \begin{bmatrix} \theta_1(t) - d_1 \cos(\alpha(t) - \alpha_d(t)) \\ d_1 \sin(\alpha(t) - \alpha_d(t)) \end{bmatrix}, \tag{15}$$

where G is as defined in (13).

Proof: Under the assumptions, the output of the plant is given by

$$\begin{aligned} y(t) &= P_R\theta_1(t)\cos(\alpha(t)) - P_I\theta_1(t)\sin(\alpha(t)) \\ &- P_R d_1 \cos(\alpha_d(t)) + P_I d_1 \sin(\alpha_d(t)). \end{aligned} \tag{16}$$

Keeping only the low-frequency components of the signals y_1 and y_2, we find that

$$\begin{aligned} y_1(t) &= \frac{1}{2}P_R\theta_1(t) - \frac{1}{2}P_R d_1 \cos(\alpha(t) - \alpha_d(t)) \\ &- \frac{1}{2}P_I d_1 \sin(\alpha(t) - \alpha_d(t)), \\ y_2(t) &= \frac{1}{2}P_I\theta_1(t) + \frac{1}{2}P_R d_1 \sin(\alpha(t) - \alpha_d(t)) \\ &- \frac{1}{2}P_I d_1 \cos(\alpha(t) - \alpha_d(t)), \end{aligned} \tag{17}$$

and the result is obtained.

Comments: The elimination of the high-frequency components within the system can be achieved by proper low-pass filtering of the signals $y_1(t)$ and $y_2(t)$. In the algorithm discussed in this paper, the signals are applied to a compensator $C(s)$ which is low-pass in nature. Although the filtering is far from ideal, simulations show that our approximations are satisfactory for the compensator design without the need for additional filtering.

3.3 Compensator Design

Equation (15) can be viewed as an alternative description of the plant, with two inputs θ_1 and α and two outputs y_1 and y_2. Although this equation is nonlinear, a linear system is obtained if the phase error $\alpha - \alpha_d$ is small. This system is described by

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = G \begin{bmatrix} \theta_1(t) - d_1 \\ d_1(\alpha(t) - \alpha_d(t)) \end{bmatrix}. \tag{18}$$

Using this linearized result, several methods for designing $C(s)$ can be used. Our approach, while not the best or the most sophisticated, yields a simple design for implementation purposes. We define two variables x_1 and x_2 as

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = G^{-1} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \tag{19}$$

so that

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \theta_1(t) - d_1 \\ d_1 \left(\int_0^t (\theta_2(\sigma) - \omega_1) d\sigma + \alpha(0) - \alpha_d(0)\right) \end{bmatrix}. \tag{20}$$

As a result, the dynamics of the system from the parameters θ_1 and θ_2 to the variables x_1 and x_2 are decoupled from one another and are those of a gain of 1 and of an integrator with a gain d_1, respectively. The unknown parameters d_1, ω_1, $\alpha_d(0)$ act as constant disturbances. The compensator $C(s)$ may then be designed as the cascade of the transformation (19) and control laws of the form

$$\begin{aligned} \theta_1 &= \frac{C_1(s)}{s}[x_1], \\ \theta_2 &= \frac{C_2(s)}{s}[x_2], \end{aligned} \tag{21}$$

where the integrators are included to reject the disturbances composed of d_1 and $\alpha_d(0)$. The transfer functions $C_1(s)$ and $C_2(s)$ are designed to guarantee the closed-loop stability of the two systems $P_1(s) = 1$ and $P_2(s) = d_1/s$. One possible choice is

$$C_1(s) = -g_1, \qquad C_2(s) = -g_2 \frac{s+a}{s+b}. \tag{22}$$

Because the magnitude of the disturbance d_1 acts as a gain in the second transfer function, the parameters of the compensator $C_2(s)$ must be designed for a range of magnitudes of the disturbance d_1. Simple techniques can also be used to estimate the disturbance level before the system is turned on.

The algorithm requires the knowledge of the matrix G, which depends on ω_1. Similar to the indirect algorithm, one can resolve this problem in two ways. One may choose to design a compensator that works satisfactorily for a range of matrices G corresponding to a range of frequencies of the unknown disturbance. Often, the uncertainty in the frequency of the disturbance is small enough such that variations in the G matrix are inconsequential, yielding adequate performance for a compensator designed for a mid-range disturbance frequency. Alternatively, one could use for G the equivalent transfer function matrix corresponding to the estimated frequency $\dot{\alpha} = \theta_2$. In either case, knowledge of the frequency response of the plant in the frequency range of interest is required.

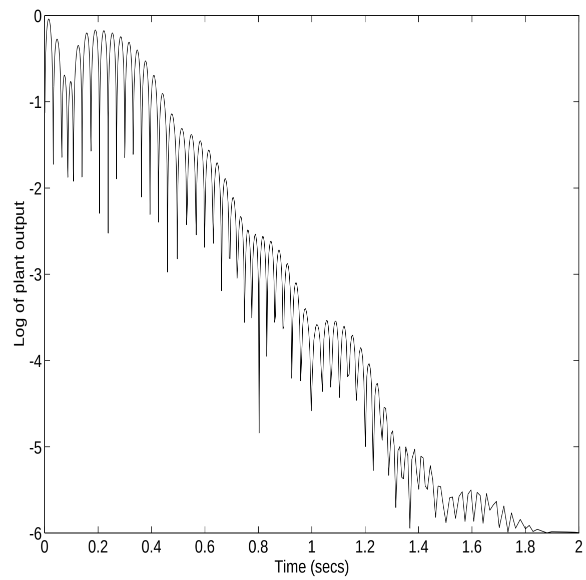

Figure 6: Log of the plant output for the direct algorithm

3.4 Simulations of the Direct Algorithm

We now explore the performance of the direct algorithm via simulation. We consider the same situation as in the indirect algorithm, in which $P(s) = 100/(s + 100)$, $\omega_1 = 100$, and $d_1 = 1$. For the direct algorithm, the parameter g_1 is set to 10, leading to a closed-loop pole for the first system at -10 rad/s. The other parameters are set to $g_2 = 400$, $a = 5$, and $b = 30$, leading to three closed-loop poles for the second system located at -10 rad/s and $-10 \pm j10$ rad/s. The initial states of all parameters are zero except for $\theta_1(0) = 0.9$, $\theta_2(0) = 90$ rad/s, and $\alpha(0) = 90$ degrees. Note that, since $\alpha_d(0) = 0$, the system is initialized

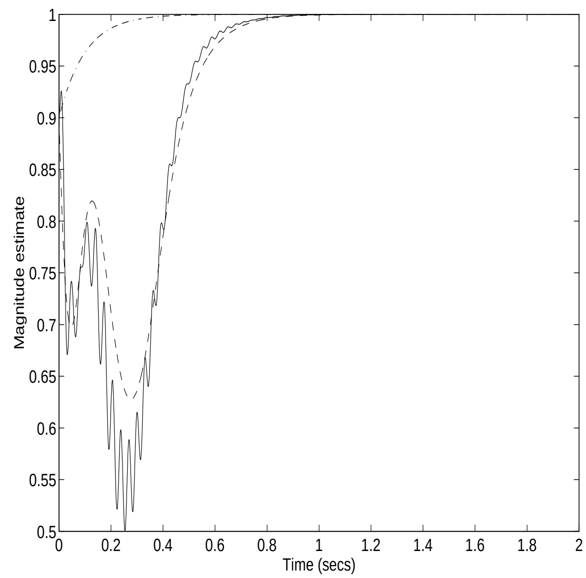

Figure 7: Magnitude estimate for the direct algorithm

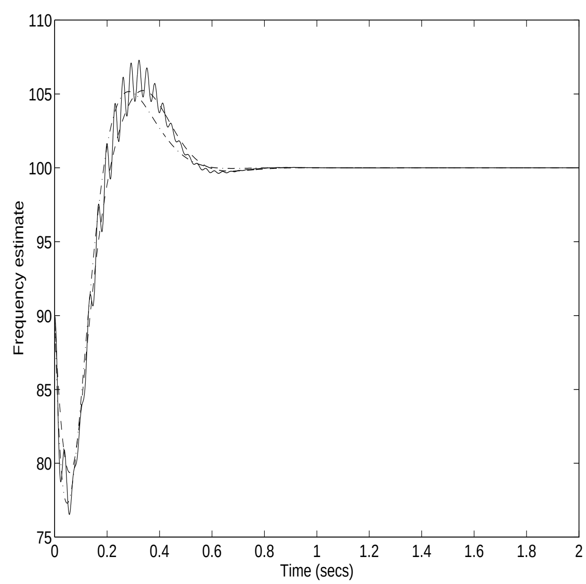

Figure 8: Frequency estimate for the direct algorithm

with a large phase error, such that the linearized analysis is less accurate.

Fig. 6 shows the log of the output of the plant. The output is found to decrease to negligible values in less than two seconds. The transient behavior of the magnitude estimate θ_1 is shown in Fig. 7, where the solid line is the parameter response, the dashed line is the response predicted using the nonlinear approximation in (15), and the dot-dashed line is the response predicted using the linear approximation in (18). The equivalent behaviors of the actual and theoretically-predicted values of the frequency estimate θ_2 are shown in Fig. 8. Note that the matches between the theoretical and actual behaviors of the estimates are particularly good when using the nonlinear approximation, whereas the analysis using the linear approximation is only accurate for the frequency estimate. Although the nonlinear effects are clearly significant, our design based on the linear approximation provides adequate convergence of the system.

In these simulations, the system converged despite a 10% initial error in frequency. It was found that the scheme was able to acquire frequencies with errors up to 30% in this case. It is well-known that an important characteristic of phase-locked loops is their *lock-in* (or *pull-in)* range (Hambley, 1990). Careful design of the loop transfer function can increase the lock-in range and improve performance. Additional supervisory logic might also be used for acquisition, and would be part of a practical design. In addition, although the elements of G were set to their nominal values corresponding to a disturbance frequency of 100 rad/s for these plots, simulations with the nominal G replaced by a continuously-adjusted matrix according to the estimated frequency of the disturbance did not exhibit significant differences in transient behaviors. Other simulations with slowly-varying disturbances and with measurement noises indicate that the direct algorithm performs well in these situations, and the effects of these variations can be analyzed precisely using the linear approximation, as described in Bodson and Douglas (1996) and Bodson (1996).

4 Conclusions

In this paper, we have presented two methods for the rejection of sinusoidal disturbances with unknown frequency. While disturbance rejection for signals with known frequency has been addressed extensively in the literature using various approaches, the analogous task for signals with unknown frequency has received comparably little attention despite its importance in applications such as active noise and vibration control. Our study of an algorithm for the determination of the frequency of an unknown signal yielded an averaging analysis that was useful for system design. It was shown that this frequency estimation algorithm could be combined with an AFC algorithm to obtain an indirect algorithm for the cancellation of disturbances with unknown frequency. While this system was able to lock on a sinusoidal signal with no *a priori* information about the frequency of the disturbance, the convergence properties of the scheme were found to be less than ideal. A second algorithm incorporating a phase-locked loop within the cancellation scheme yielded improved convergence properties, and our averaging analysis provided a precise prediction of the dynamic characteristics of the system. A successful scheme for sinusoidal disturbance cancellation scheme would combine the direct algorithm presented here with an initialization scheme for providing rough initial estimates of the disturbance frequency. Simulations verified the results of the analysis and indicated the usefulness of the algorithms for the sinusoidal disturbance cancellation task.

References

R. Shoureshi and T. Knurek, "Automotive Applications of a Hybrid Active Noise and Vibration Control," *IEEE Control Systems,* vol. 16, no. 6, pp. 72-78, 1996.

M. Bodson, "Cancellation of Sinusoidal Disturbances with Unknown Frequency," *Proc. of the Ninth Yale Workshop on Adaptive and Learning Systems,* New Haven, CT, pp. 174-179, 1996.

M. Bodson and S.C. Douglas, "Adaptive Algorithms for the Rejection of Periodic Disturbances with Unknown Frequency," *Proc. of the 13th World Congress of the International*

Federation of Automatic Control, San Francisco, CA, vol. K, pp. 229-234, 1996.

M. Bodson, A. Sacks and P. Khosla, "Harmonic Generation in Adaptive Feedforward Cancellation Schemes," *IEEE Trans. on Automatic Control*, vol. 39, no. 9, 1994, pp. 1939-1944.

U. Emborg and C.F. Ross, "Active Control in the Saab 340," *Proc. Second Conf. Recent Adv. Active Contr. Sound Vib.,* Supplement, pp. S67-S73, Blacksburg, VA, April 1993.

L.J. Eriksson, "A Practical System for Active Attenuation in Ducts," *Sound and Vibration,* vol. 22, no. 2, pp. 30-34, February 1988.

A.R. Hambley, *An Introduction to Communication Systems*, W.H. Freeman and Co., New York, 1990.

W. Messner and M. Bodson, "Design of Adaptive Feedforward Algorithms Using Internal Model Equivalence," *International Journal of Adaptive Control and Signal Processing,* vol. 9, pp. 199-212, 1995.

P.A. Nelson and S.J. Elliott, *Active Control of Sound,* Academic Press, London, England, 1992.

P.A. Regalia, "An Improved Lattice-Based Adaptive IIR Notch Filter," *IEEE Trans. on Signal Processing*, vol 39, no. 9, 1991, pp. 2124-2128.

A. Sacks, M. Bodson, and P. Khosla, "Experimental Results of Adaptive Periodic Disturbance Cancellation in a High Performance Magnetic Disk Drive," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 118, pp. 416-424, 1996.

A. Sacks, M. Bodson, and W. Messner, "Advanced Methods for Repeatable Runout Compensation," *IEEE Trans. on Magnetics,* vol. 31, no. 2, pp. 1031-1036, 1995.

S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness,* Prentice-Hall, Englewood Cliffs, NJ, 1989.