Equivalence between Adaptive Feedforward Cancellation and Disturbance Rejection Using the Internal Model Principle*

Xiuyan Guo† and Marc Bodson‡

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Abstract: The paper shows that a large class of adaptive algorithms for disturbance cancellation yields control systems that are equivalent to compensators implementing the internal model principle (IMP). The fact has been known to be true for periodic disturbances with fixed frequency. However, the paper shows that the result can be extended to disturbances of time-varying frequency (*i.e.,* frequency-modulated signals), regardless of the rate of variation of the frequency. In particular, several adaptive controllers are shown to be equivalent to linear time-varying compensators implementing the IMP. Further, a pseudo-gradient algorithm produces the same responses as a polytopic linear parameter-varying compensator.

Keywords: adaptive control, augmented error, adaptive feedforward cancellation, periodic disturbances, internal model principle, linear parameter-varying systems.

1 Introduction

The paper considers the problem of rejecting sinusoidal disturbances with time-varying frequency, *i.e.,* signals of the form

$$d(t) = \theta_c^* \cos(\alpha_d(t)) + \theta_s^* \sin(\alpha_d(t)), \qquad \dot{\alpha}_d(t) = \omega_d(t) \tag{1}$$

The definition of the disturbance is comparable to the definition of a frequency-modulated signal in communication systems. The time-varying frequency $\omega_d(t)$ is called the *instantaneous frequency* of the signal $d(t)$. $\omega_d(t)$ is assumed to be bounded and known, and the parameters θ_c^* and θ_s^* are assumed to be constant and unknown. Although the parameters θ_c^* and θ_s^* may vary in some

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†X. Guo is with QSecure, 333 Distel Cir, Los Altos, CA 94022, USA (email: xguo@qsecure.com).

‡M. Bodson is with the Department of Electrical and Computer Engineering, University of Utah, 50 S Central Campus Dr Rm 3280, Salt Lake City, UT 84112, USA (email: bodson@ece.utah.edu).

applications at the same rate as the frequency, there is a class of problems where they can be considered constant or slowly-varying. These problems include those associated with eccentricity and non-circularity of rotating devices. Eccentricity compensation is discussed in [5]. In compact disc players, the frequency of rotation decreases as music is being played, so that eccentricity causes a periodic disturbance of varying frequency [12]. Another example arises in web transport systems, such as the paper machine considered in [13]. Experiments on the testbed of [13] showed that the spectrum of the web tension had large components at the frequencies of rotation of the winding and unwinding rolls, due to the eccentricity and the non-circularity of the rolls. The frequencies varied as paper was transferred from one roll to another at high speed.

The paper considers a class of adaptive feedforward cancellation algorithms that can be applied to reject such frequency-modulated disturbances. The main result of the paper is that the algorithms are exactly equivalent to a set of compensators implementing the internal model principle. The equivalence is very general and, in the case of several standard adaptive algorithms, it yields linear time-varying compensators implementing the internal model principle (IMP). Interestingly, the proof of equivalence for the time-varying frequency case is no more complicated than the earlier proof that assumed a fixed frequency and could not be extended to the time-varying case. Further, the new proof is more general, by accounting exactly for arbitrary initial conditions. The equivalence provides an opportunity to apply knowledge gained either from adaptive control or from robust linear control to the other field. For example, the robustness of adaptive systems can be assessed in previously unknown ways using the equivalent linear system. On the other hand, the adaptive theory provides interesting control algorithms, such as those based on an augmented error and on least-squares algorithms, which would not intuitively be found by using a linear control approach based on the internal model principle. The adaptive implementation also enables the direct use of angular measurements, without reconstruction of the instantaneous frequency of the disturbance. Although omitted from the paper for brevity and clarity, extension of the result to disturbances with multiple sinusoidal disturbances is straightforward, and yields compensators with multiple parallel paths, each similar to the single sinusoid case.The paper concludes with examples designed to illustrate the application of the theoretical results.

2 Adaptive Feedforward Cancellation (AFC)

We consider the disturbance rejection problem for a system described by

$$y(t) = p(t) * (u(t) - d(t)) \tag{2}$$

where $*$ denotes the convolution operation and $p(t)$ is the impulse response of the system, or plant. The plant is assumed to be linear time-invariant with transfer function $P(s)$. The signals $y(t)$, $u(t)$, and $d(t)$ are the plant output, the control input, and the disturbance signal, respectively. The plant will be assumed to be stable for the application of the AFC scheme (not for the equivalence result). If the plant is unstable, a stabilizing controller can be designed and the techniques can be applied to the closed-loop system. The goal of the control system is to generate a control input $u(t)$ such that $y(t) \to 0$ as $t \to \infty$.

Given a disturbance of the form (1), the control input can be chosen to be

$$u(t) = \theta_c(t)\cos(\alpha_d(t)) + \theta_s(t)\sin(\alpha_d(t)) \tag{3}$$

so that disturbance cancellation could, in theory, be achieved exactly by letting $\theta_c(t) = \theta_c^*$, $\theta_s(t) = \theta_s^*$. Since the nominal parameters are unknown, the control strategy is to use adaptation so that the parameters converge to values such that the disturbance is rejected.

Let the nominal and adaptive parameter vectors be

$$\theta^* = \begin{pmatrix} \theta_c^* & \theta_s^* \end{pmatrix}^T \qquad \theta(t) = \begin{pmatrix} \theta_c(t) & \theta_s(t) \end{pmatrix}^T \tag{4}$$

The *regressor vector* is defined to be

$$w(t) = \begin{pmatrix} \cos(\alpha_d(t)) & \sin(\alpha_d(t)) \end{pmatrix}^T \tag{5}$$

Therefore

$$d(t) = w^T(t)\theta^*, \;\; u(t) = w^T(t)\theta(t) \tag{6}$$

and

$$y(t) = p(t) * (u(t) - d(t)) = p(t) * \left(w^T(t)\,(\theta(t) - \theta^*)\right) \tag{7}$$

Equation (7) falls into the framework of adaptive control theory [11], and several algorithms are available to update the adaptive parameters. Commonly-used adaptive algorithms include the *pseudo-gradient*, *gradient* and *augmented error* (AE) algorithms. We briefly review these algorithms.

2.1 AFC with Pseudo-gradient Algorithm

The pseudo-gradient algorithm is simply given by

$$\dot{\theta}(t) = -g w(t) y(t) \tag{8}$$

where $g > 0$ is the adaptation gain. If the plant transfer function was the identity, this would be a gradient algorithm for the problem, hence the name pseudo-gradient. Adaptive control theory indicates that Lyapunov stability of the overall system and convergence of the error $y(t)$ to zero is ensured if the plant transfer function $P(s)$ is *strictly positive real* (SPR) [11]. However, the SPR condition is rarely, if ever, satisfied in practice.

2.2 AFC with Gradient Algorithm

The adaptive parameters in a gradient algorithm are updated according to

$$\dot{\theta}(t) = -g w_F(t) y(t) \tag{9}$$

$$w_F(t) = \hat{p}(t) * w(t) \tag{10}$$

where $\hat{p}(t)$ is an estimate of the plant impulse response $p(t)$ and $w_F(t)$ is called the *filtered regressor vector*. Note that the algorithm follows the direction of the gradient under the assumption of slowly-varying parameters. It is known that the gradient algorithm is generally not stable for all adaptation gains $g > 0$. However, under the condition that $\hat{p}(t) = p(t)$, the stability of the gradient algorithm can be proved using an averaging analysis for small gain [2].

2.3 AFC with Augmented Error Algorithm

Instead of using $y(t)$ to update the adaptive parameters in the gradient algorithm as shown in (9), the augmented error algorithm is obtained by using an augmented error

$$e_a(t) = y(t) + w_F^T(t)\theta(t) - u_F(t) \tag{11}$$

where

$$u_F(t) = \hat{p}(t) * u(t) \tag{12}$$

and $w_F(t)$ is the filtered regressor vector defined in (10). The augmented error algorithm is stable for all $g > 0$ and the error converges to zero, assuming that the plant is exactly known (without requiring an SPR condition on the plant). Exponentially convergence of the adaptive parameters is achieved if $w_F(t)$ is PE [1].

3 Equivalence between AFC Algorithms and LTV Compensators Implementing the IMP

The main result of this paper is the equivalence between the AFC algorithms and time-varying compensators implementing the internal model principle. The equivalence is shown graphically in Fig. 1, where $R(t)$ is a matrix specified in the following fact.

Fact: let an AFC algorithm be given by

$$\begin{aligned} u(t) &= w^T(t)\theta(t) \\ w(t) &= \begin{pmatrix} \cos(\alpha_d(t)) & \sin(\alpha_d(t)) \end{pmatrix}^T \end{aligned}$$

Figure 1: General LTV IMP equivalence of an AFC algorithm

with arbitrary update law

$$\dot{\theta}(t) = F\left(y(\cdot), w(\cdot)\right), \qquad \theta(0) = \theta_0$$

where $F\left(y(\cdot), w(\cdot)\right)$ is some causal operator describing the adaptation mechanism. Then, the operator from $\dot{\theta}(t)$ to $u(t)$ is equivalent to a linear time-varying system described by the state-space realization

$$\begin{aligned} \dot{x}(t) &= A_d(t)x(t) + R(t)F\left(y(\cdot), w(\cdot)\right), \qquad x(0) = R(0)\theta_0 \\ u(t) &= C_d x(t) \end{aligned}$$

where

$$\begin{aligned} A_d(t) &= \begin{bmatrix} 0 & \omega_d(t) \\ -\omega_d(t) & 0 \end{bmatrix}, \quad C_d = \begin{bmatrix} 0 & 1 \end{bmatrix} \\ R(t) &= \begin{bmatrix} \sin(\alpha_d(t)) & -\cos(\alpha_d(t)) \\ \cos(\alpha_d(t)) & \sin(\alpha_d(t)) \end{bmatrix}, \quad \omega_d(t) = \frac{d\alpha_d(t)}{dt} \end{aligned}$$

Proof: The proof uses the linear transformation $x(t) = R(t)\theta(t)$, which is well defined for all $t \geq 0$, and its inverse $R^{-1}(t)$, which is also well-defined since $R^{-1}(t) = R^T(t)$. Given

$$\dot{R}(t) = \omega_d(t) \begin{bmatrix} \cos\alpha_d(t) & \sin(\alpha_d(t)) \\ -\sin(\alpha_d(t)) & \cos(\alpha_d(t)) \end{bmatrix}$$

it follows that

$$\begin{aligned} \dot{x}(t) &= \frac{d}{dt}(R(t)\theta(t)) = \dot{R}(t)R^{-1}(t)x(t) + R(t)\dot{\theta}(t) \\ u(t) &= w^T(t)\theta(t) = w^T(t)R^{-1}(t)x(t) \end{aligned}$$

The fact is obtained by checking that

$$\dot{R}(t)R^{-1}(t) = A_d(t), \quad w^T(t)R^{-1}(t) = C_d$$

□

The model of the disturbance obtained in the above theorem is

$$\dot{x}(t) = A_d(t)x(t), \quad d(t) = C_d x(t) \tag{13}$$

and it can be easily shown that the disturbance in (1) is the solution of the nonautonomous system (13) with initial conditions $x(0) = \begin{bmatrix} -\theta_s^* & \theta_c^* \end{bmatrix}^T$. In other words, the state-space system (13) is a model of the frequency-modulated disturbance, as required by the internal model principle [7].

The result of the above fact was known to be true for some time in the fixed frequency case [9], and interesting consequences were observed [4], [1], [10]. Preliminary discussion of the time-varying case was first presented in [3].

3.1 LTV IMP Equivalence for Pseudo-gradient Algorithm

We now turn to the specialization of the results to the AFC algorithms that were presented earlier. The pseudo-gradient algorithm described by (6) and (8) is equivalent to the LTV controller $\Sigma(t)$ with state-space realization

$$\begin{aligned} \dot{x}(t) &= A_d(t)x(t) - \begin{bmatrix} 0 & g \end{bmatrix}^T y(t) \\ u(t) &= C_d x(t) \end{aligned} \tag{14}$$

The result can be checked from the fact, given that

$$F\left(y(\cdot), w(\cdot)\right) = -gw(t)y(t), \quad R(t)w(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$$

The equivalence can be used for a number of purposes. Stability properties of the adaptive algorithm can be determined for plants that are not SPR. In Appendix, it is shown that, because the parameter $\omega_d(t)$ appears linearly in the LTV equivalent controller (14), the closed-loop system belongs to the class of *polytopic linear parameter-varying* (PLPV) systems. Such systems are nonautonomous LPV systems such that

$$\dot{x}(t) = A(\pi(t))x(t)$$

where $A(\cdot)$ is an affine function of a time-varying parameter $\pi(t)$ and $\pi(t)$ varies in a polytopic set

Π with vertices $\xi_1, \xi_2, \cdots, \xi_N$, that is

$$\begin{aligned}\Pi \;:\; &= Co\{\xi_1, \xi_2, \cdots, \xi_N\} \\ &= \left\{\sum_{i=1}^{N} \alpha_i(t)\xi_i :\; \alpha_i(t) \geq 0, \sum_{i=1}^{N} \alpha_i(t) = 1\right\}\end{aligned}$$

Stability of the closed-loop adaptive system can be ensured if quadratic stability of the PLPV system can be proved. In turn, this property can be guaranteed if there exists a single positive definite matrix P such that

$$A_i^T P + PA_i < 0,\; i = 1, 2$$

for two matrices given in Appendix. Such equation can be solved using a *linear matrix inequality* (LMI) solver [6].

If the controller is designed so that the closed-loop system is a uniformly exponentially stable linear time-varying system, the trajectories of the system will always be bounded, even if the frequency of the disturbance is not known exactly (since the disturbance signal is bounded). In the time-invariant case [4], it has been found that the parameters of the adaptive system varied at a frequency equal to the difference between the algorithm and disturbance frequencies. A similar result is expected to hold in the time-varying case, although the tracking error may be as conveniently or precisely quantified as is possible in the time-invariant case.

3.2 LTV IMP Equivalence for Gradient Algorithm

The AFC with the gradient algorithm in (9) is equivalent to an LTV controller $\Sigma(t)$ with

$$\dot{x}(t) = A_d(t)x(t) - gB(t)y(t), \quad u(t) = C_d x(t) \tag{15}$$

where

$$B(t) = R(t)w_F(t) \tag{16}$$

It should be noted that the LTV equivalent controller in (15) is not an LPV system any more, since $B(t)$ is not linearly dependent on $\omega_d(t)$. Therefore, the ability to use linear time-varying system theory results is considerably reduced.

3.3 LTV IMP Equivalence for AE Algorithm

The adaptive system with AE algorithm is equivalent to the LTV system $\Sigma(t)$ with

$$\dot{x}(t) = A_c(t)x(t) + B_c(t)\left(y(t) - u_F(t)\right), \quad u(t) = C_d x(t) \tag{17}$$

where $u_F(t)$ and $B(t)$ were defined in (12), (16), and

$$A_c(t) = A_d(t) - gB(t)B^T(t), \quad B_c(t) = -gB(t) \tag{18}$$

3.4 Application of the Results

The results enable the transfer of knowledge between the areas of adaptive systems and robust linear control. From the adaptive system implementation, one gains a number of algorithms that would not intuitively emerge from linear control theory. In particular, the augmented error structure provides an algorithm that is guaranteed to be stable (Lyapunov stable in general, and exponentially stable under PE conditions) no matter what the range and rate of variation of the frequency parameter are [1]. Least-squares algorithm can also replace gradient algorithms, yielding an additional set of time-varying compensators unlikely to be obtained by other means. The adaptive implementation of the algorithms is also useful in cases where the angle α_d can be directly measured, such as through the use of an encoder on the device that causes the disturbance. The AFC algorithms do not require the frequency to be computed from the angular measurements.

On the other hand, the equivalence to linear controllers enables one to obtain robustness estimates that would normally not be available from adaptive control theory. A model of plant uncertainty is the multiplicative uncertainty

$$P(s) = \hat{P}(s)(1 + w_I(s)\Delta(s)) \tag{19}$$

where $w_I(s)$ is some known stable transfer function and $\Delta(s)$ can be any stable transfer function whose magnitude is less than or equal to one in the frequency domain (*i.e.,* $\|\Delta(j\omega)\|_\infty \leq 1$). The closed-loop LTV system including the uncertainty is shown in Fig. 2, where $\Sigma(t)$ is defined by (14), (15) or (17) for the pseudo-gradient, gradient, and AE algorithms, respectively. If the applicable vector $w(t)$ or $w_F(t)$ is persistently exciting, the nominal closed-loop system (*i.e.,* with $\Delta(s) = 0$) is known to be exponentially stable. From the small gain theorem, robust stability of the system can be guaranteed if the *root mean square* (RMS) gain from $y_\triangle(t)$ to $u_\triangle(t)$ is less than 1. This approach will provide an estimate of the robustness margin, although it may be difficult to compute and quite conservative. For the pseudo-gradient algorithm, the system from $y_\triangle(t)$ to $u_\triangle(t)$ is a PLPV system, so that the robust performance can be evaluated using the LMI control toolbox [8].

4 Simulation Results

In this section, we give an example of application of the results by considering the linear two-mass-spring-damper system of [6]. The system is shown in Fig. 3. In Fig. 3, $d(t)$ is the external sinusoidal disturbance force acting on the second mass, and the output of the system $y(t)$ is the acceleration of the second mass, or $\ddot{x}_2(t)$. Here, we consider the case where the control signal (force) is $u_2(t)$

Figure 2: Robust stability of AFC algorithm with multiplicative plant uncertainty

and acts on the second mass, *i.e.* the control signal and the disturbance act at the same location.

Figure 3: Two-mass-spring-damper system

The plant parameters are selected to be the same as in [6] where $m_1 = m_2 = 1$, $k_1 = k_2 = 100$, $c_1 = c_2 = 1$. Then, the corresponding plant is

$$P(s) = \frac{s^2(s^2 + 2s + 200)}{(s^2 + 0.382s + 38.2)(s^2 + 2.618s + 261.8)} \tag{20}$$

The objective is to design robust AFC algorithms with fast convergence rate to eliminate the effect of the disturbance $d(t)$. The frequency is assumed to vary between $\omega_{\min}$ and $\omega_{\max}$ and $d(t)$ has constant magnitude equal to 1.

The only designed parameter in the AFC algorithms is the adaptation gain g, which can be constant or made a function of the frequency. In general, the convergence rate of the algorithms is improved if the gain g is increased. Therefore, a simple design objective is to maximize the adaptation gain while maintaining a certain level of robustness to uncertainties in the plant model.

Here, we consider the pseudo-gradient algorithm to cancel a periodic disturbance with time-varying frequency decreasing linearly from $\omega_{\max} = 33$ rad/s to $\omega_{\min} = 21$ rad/s in the first 4 seconds of the simulation and increasing the other way during the following 4 seconds. Note that the plant transfer function is *not* strictly positive real, so that Lyapunov analysis of the adaptive system would not support the use of the algorithm. However, the LTV equivalence with a limited frequency range enables a design with guaranteed stability properties, further accounting for possible unmodeled dynamics. A fixed adaptation gain will keep the adaptive system stable for any frequency in the stated range, any rate of change in instantaneous frequency, and any uncertainty within the bounds.

Figure 4: Plant output with pseudo-gradient algorithm

To illustrate the application of the theory, we arbitrarily select a multiplicative plant uncertainty $w_I(s) = \left(\frac{1}{80}s + 0.1\right) / \left(\frac{1}{4\times 80}s + 1\right)$. Note that a plant of the form $P(s) = \frac{\sigma}{s+\sigma}\hat{P}(s)$ fits the uncertainty model for $\sigma \geq 75$. Using the LMI control toolbox with the PLPV system, one finds that the largest adaptation gain that will ensure robust stability in the presence of the multiplicative uncertainty is $g = 1.1$. The RMS gain from $y_\Delta(t)$ to $u_\triangle(t)$ of Fig. 2 is 0.9990. The result of a simulation with $g = 1.1$ is shown in Fig. 4, which indicates that the disturbance is rejected well despite the rapidly varying frequency and a non-SPR plant. The application of the LTV equivalence with the PLPV theory yields a powerful result of robust stability of the adaptive algorithm in the presence of arbitrarily fast frequency variations and uncertainty, which could not be obtained from standard adaptive control theory.

Unfortunately, the LPV theory does not apply to the other algorithms, which are not linearly parameterized in the frequency. More unfortunate still is the fact that the gradient and augmented error algorithms exhibit, in general, faster convergence and greater robustness properties than the pseudo-gradient [10]. As an alternative, one may apply the LTI equivalence assuming that the frequency varies slowly within some range. In this way, one may obtain, frequency by frequency, the optimal adaptation gain that will ensure robust stability. In practical implementation, the values of g for intermediate frequencies can be computed by linear interpolation.

5 Conclusions

Adaptive feedforward cancellation schemes were considered for the rejection of a sinusoidal disturbance with time-varying frequency. General adaptive systems were shown to be equivalent to time-varying compensators incorporating the internal model principle, without any requirement of stability, zero initial conditions, or limited range or rate of variation of the frequency parameter. For some standard adaptive algorithms, the time-varying compensators were found to be linear. The framework enabled the evaluation of the robustness properties of the AFC algorithms in ways that have not been possible without the equivalence results. Simulations illustrated the application of the theoretical results.

6 Appendix

In this appendix, we show that if the time-varying frequency $\omega_d(t)$ varies in a range $[\omega_{\min}, \omega_{\max}]$, the closed-loop system with pseudo-gradient algorithm fits the class of PLPV systems. Letting

$$\tilde{x}(t) = x(t) - x_{ss}(t), \quad \tilde{u}(t) = u(t) - d(t)$$

where $x_{ss}(t)$ is defined as

$$x_{ss}(t) = \begin{bmatrix} -\theta_s^* \cos(\alpha_d(t)) + \theta_c^* \sin(\alpha_d(t)) \\ d(t) \end{bmatrix} = R(t)\theta^*$$

Denote $B_d = \begin{bmatrix} 0 & g \end{bmatrix}^T$, so that

$$\frac{d\tilde{x}(t)}{dt} = A_d(t)\tilde{x}(t) - B_d y(t), \quad \tilde{u}(t) = C_d \tilde{x}(t)$$

Then, given a state-space realization of the plant

$$\begin{aligned} \dot{x}_p(t) &= A x_p(t) + B(u(t) - d(t)) \\ y(t) &= C x_p(t) + D(u(t) - d(t)) \end{aligned}$$

the closed-loop system is described by

$$\begin{aligned} \begin{pmatrix} \frac{dx_p(t)}{dt} \\ \frac{d\tilde{x}(t)}{dt} \end{pmatrix} &= \begin{bmatrix} A & BC_d \\ -B_d C & A_d(t) - B_d D C_d \end{bmatrix} \begin{pmatrix} x_p(t) \\ \tilde{x}(t) \end{pmatrix} \\ &= \left\{ (1-\beta(t)) \begin{bmatrix} A & BC_d \\ -B_d C & A_{d,\min} - B_d D C_d \end{bmatrix} + \beta(t) \begin{bmatrix} A & BC_d \\ -B_d C & A_{d,\max} - B_d D C_d \end{bmatrix} \right\} \begin{pmatrix} x_p(t) \\ \tilde{x}(t) \end{pmatrix} \\ &\triangleq \{(1-\beta(t))A_1 + \beta(t)A_2\} \begin{pmatrix} x_p(t) \\ \tilde{x}(t) \end{pmatrix} \end{aligned}$$

where

$$\beta(t) = \frac{\omega(t) - \omega_{\min}}{\omega_{\max} - \omega_{\min}}, \; A_{d,\min} = \begin{bmatrix} 0 & \omega_{\min} \\ -\omega_{\min} & 0 \end{bmatrix}, \text{ and } A_{d,\max} = \begin{bmatrix} 0 & \omega_{\max} \\ -\omega_{\max} & 0 \end{bmatrix}$$

Thus, the closed-loop system is a PLPV system with two vertices $\omega_{\min}$ and $\omega_{\max}$.

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