**Lecture 3: Terminated Lossless Transmission Lines**

**Text Section: 2-3**

**Portfolio:**

1) **Describe the fields on a terminated lossless transmission line, including reflections and standing waves. What is the reflection coefficient and VSWR?**

**Describe how to design a quarter-wave transformer. (Include a design example for a complex load impedance.)**

## Wave Propagation in Lossless Transmission line:

VIEWGRAPH Ulaby Table 2-2.

$\alpha$, $\beta$ , $\gamma$ , $v_p$ do not depend on shape of transmission line! (for lossless TEM lines)

But Zo does . Zo is strictly real for lossless lines (no phase change).

So, once started V,I propagate the same way. Zo controls relative magnitudes of V,I.

## Reflected Wave :

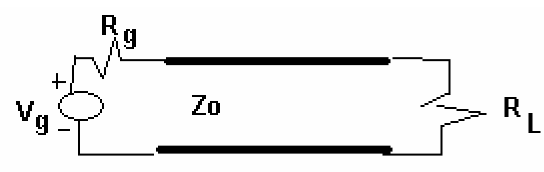
$V(z) = Vo^+ e^{-j\beta z} + Vo^- e^{j\beta z}$

$I(z) = (Vo^+ /Zo) e^{-j\beta z} - (Vo^- /Zo) e^{j\beta z}$

To find the complete solution, we need to find the voltages of the +z and –z traveling waves.

+z traveling wave comes from the generator

-z traveling wave is reflected from the load.




$Z_L = V_L / I_L$

$V_L = V(z=0) = Vo^+ + Vo^-$

$I_L = I(z=0) = Vo^+ / Zo - Vo^- / Zo$

$Z_L = [ (Vo^+ + Vo^- ) / (Vo^+ - Vo^- ) ]\ Zo$

$Vo^- = [(Z_L – Zo) / (Z_L + Zo)]\ Vo^+$

## Voltage Reflection Coefficient:

(proportion of wave reflected)

$\Gamma = Vo^- / Vo^+ = [(Z_L – Zo) / (Z_L + Zo)]$ (ratio – unitless)

$= (Z_L /Zo -1) / (Z_L /Zo +1)$

$= |\Gamma| \angle \theta_r = |\Gamma|\, e^{j\theta r}$

$V(z) = Vo^+ \, e^{-j\beta z} + \Gamma \, Vo^+ \, e^{j\beta z}$

$I(z) = (Vo^+ / Zo) \, [\, e^{-j\beta z} - \Gamma \, e^{j\beta z} \,]$

**<u>Current Reflection Coefficient:</u>**

$Io^- / Io^+ = - Vo^- / Vo^+ = - \Gamma$

**Example:**

100-ohm line(Zo) connected to a 50-ohm termination ($R_L$)

$\Gamma = (50/100 - 1) / (50/100 + 1) = -1/3 = Vo^- / Vo^+$

For a 1V sine wave, 1/3 V sine wave is reflected back, out of phase.

Draw. ($Vo^+ = 1$ at open end)

**Example:**

100-ohm line (Zo) left open ($R_L = \infty$ )

$\Gamma = (\infty /100 - 1) / (\infty /100 + 1) = 1 = Vo^- / Vo^+$

For a 1V sine wave, 1 V sine wave is reflected back, in phase.

Draw. ($Vo^+ = 1$ at open end, sum = 2V at open end)

**Example:**

100-ohm line (Zo) with a short ($R_L = 0$)

$\Gamma = (0/100 - 1) / (0/100 + 1) = -1 = Vo^- / Vo^+$

For a 1V sine wave, 1 V sine wave is reflected back, out of phase.

Draw as function of time. ($Vo^+ = 1$ at open end, sum = 0V at open end)

What about elsewhere?

**Example:** (so how do you get rid of these reflections!)

100-ohm line (Zo) with $R_L$ = 100 ohms

$\Gamma = (100/100 - 1) / (100/100 + 1) = 0! = Vo^- / Vo^+$

For a 1V sine wave, 0 V is reflected back

Draw as function of time. ($Vo^+$ =1 at open end , sum = 1V at open end)

**<u>Standing Waves:</u>**

We have calculated what happens at one location (end of line), as a function of time. What about the rest of the line?

Sum of $Vo^-$ and $Vo^+$ creates a "standing wave".

The total wave on the transmission line is the "interference" of the two waves.

At any point, it is a sine wave (same frequency as incident).

At this point, the magnitude and phase is controlled by the sum of the two waves.

The largest magnitude is when they add in phase (max 2V for 1V incident)

The smallest magnitude is when they add out of phase (min 0V for 1V incident)

**Understanding Standing Wave Diagrams:**

The diagrams in Figure 2-14 show the ENVELOPE of the wave as a function of distance.

Inside this envelope, the summed wave is still going up and down.

Simulation.

Velocity of propagation of a pure standing wave? ZERO.

Wavelength of a standing wave? $\lambda$

Repetition period? $\lambda / 2$

**<u>Voltage Standing Wave Ratio (VSWR):</u>**

$S = |V|_{max} / |V|_{min} = (1 + |\Gamma|) / (1 - |\Gamma|)$ dimensionless

Also: $\Gamma = (S-1) / (S+1)$

S is large when there is a large reflected wave, small when there is a small reflected wave.

It measures the mismatch.

Large S is "bad" for most things.

There is a VSWR meter!

**Example:**

Open circuit

$\Gamma = 1 \angle 0$

$S = 2 / 0 = \infty$

**Example:**

Short circuit

$\Gamma = (0/100 - 1) / (0/100 + 1) = -1 = 1\angle \pi = 1 \angle 180°$

$S = 2 / 0 = \infty$

**Example:**

Perfectly matched line (no reflection)

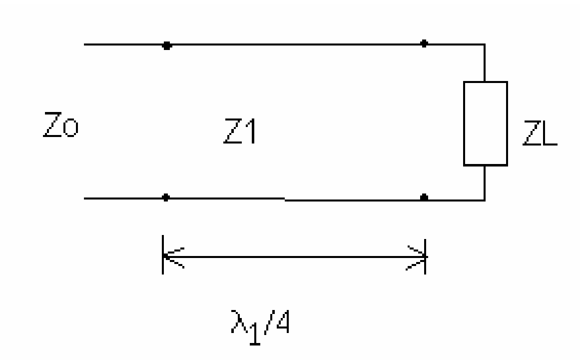
$\Gamma = (100/100 - 1) / (100/100 + 1) = 0! = Vo^- / Vo^+$

$S = 1 / 1 = 1$ ← This is the best we can do.

**Quarter Wave Transformer**

Text section 2.5

A quarter wave transformer is used to match one REAL impedance to another.




ZL: Impedance of the load (must be strictly real, no imaginary part)

Zo: characteristic impedance of the line you want to connect ZL to.

Z1: Characteristic impedance of the quarter-wave transformer you will place between ZL and Zo to match the line

$Z1 = \sqrt{(Zo\ ZL)}$

$\lambda_1$ : wavelength of the line that is used to make the quarter-wave transformer

How does this work?

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan\beta\ell}{Z_1 + jR_L \tan\beta\ell}$$

$$for \quad \beta\ell = (2\pi/\lambda)(\lambda/4) = \pi/2$$

$$Z_{in} = Z_1^2 / R_L = Zo \ (for\ match)$$

$$so\ Z_1 = \sqrt{Z_o R_L}$$

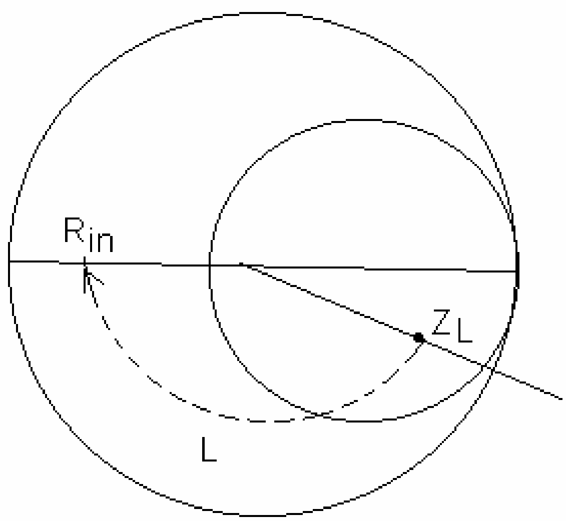
What if $Z_L$ is complex?

Add an additional length of Zo line between the load $Z_L$ and the source to make $Z_L$ strictly real.

$$R_{in} = Z_1 \frac{(R_L + jX_L) + jZ_1 \tan\beta\ell}{Z_1 + j(R_L + jX_L)\tan\beta\ell}$$

Solve for Real and Imaginary parts, set imaginary part =0 by appropriate choice of $\beta l$ .

Another way to do this: Use the Smith Chart. Rotate from the load towards the generator until you reach the REAL line.




Frequency Response of Quarter-Wave Matching:

FIGURE FROM TEXT.

This is a narrow-band matching network because the wavelength changes with frequency, so the quarter-wave section is only quarter-wave at discrete frequencies.

To make this broader-band, use multiple sections. There are many ways of designing multiple sections. We will learn about this later.