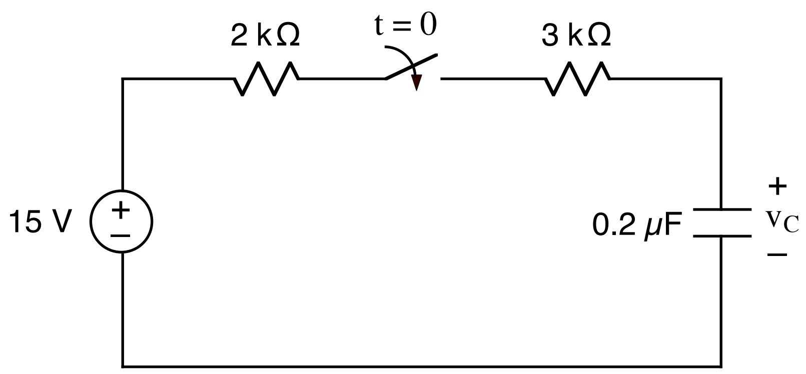
HOMEWORK #11 Solution Prob 2

Ex:



After being open for a long time, the switch closes at t = 0. $v_C(t = 0^-) = 0V$. Find $v_C(t)$ for $t > 0$.

sol'n: Use the general form of solution for RC problems.

$$v_C(t>0) = v_C(t\to\infty) + [v_C(0^+) - v_C(t\to\infty)]\, e^{-t/R_{Th}C}$$

We now proceed to find the following values:

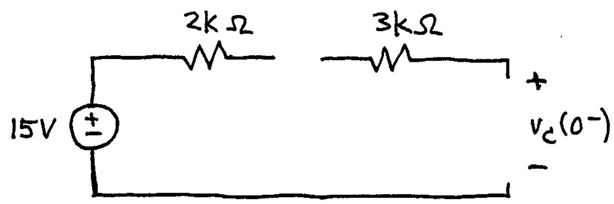
$v_C(0^+)$, $v_C(t\to\infty)$, and R_{Th}

To find $v_C(0^+)$, we consider $t=0^-$ and find $v_C(0^-)$. Since v_C is an energy variable that cannot change instantly, we have $v_C(0^+) = v_C(0^-)$.

At $t=0^-$, currents and voltages have stabilized, and all time derivatives of currents and voltages are zero.

Thus, $i_C = C\dfrac{dv_C}{dt} = C\cdot 0 = 0$. C looks like open.

$t = 0^-$: C = open , switch open



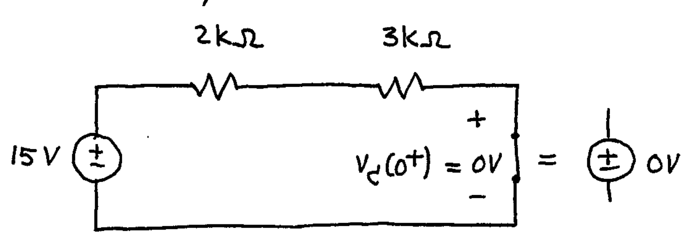
From the circuit diagram, we cannot determine $v_c(0^-)$. The C could be charged to some voltage, and it would remain at that voltage forever.

Fortunately, the problem states that $v_c(0^-) = 0V$.

$t = 0^+$: v_c cannot change instantly, so

$$v_c(0^+) = v_c(0^-) = 0V$$

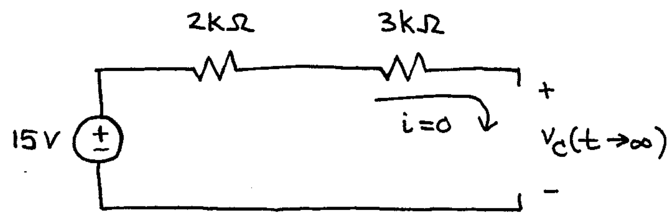
If needed a circuit model at $t = 0^+$, we would model the C as a v src with value 0V. In other words, C = wire at $t = 0^+$.



To find $v_c(t \to \infty)$, we again use the idea that currents and voltages are stable and C = open.

HOMEWORK #11 Solution Prob 2 (cont.)

$t \to \infty$: C = open, switch closed

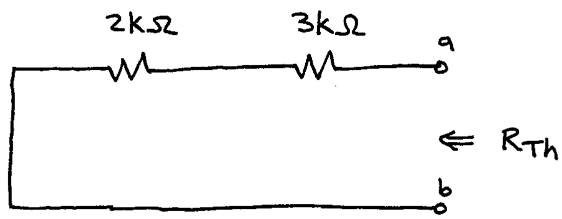


Since no current flows, the voltage drop across the 2kΩ and 3kΩ R's is 0V.
Thus, we have 15V across C:

$$v_c(t\to\infty) = 15V$$

To find R_{Th}, we remove C and find the Thevenin equivalent resistance seen looking into the terminals where C was connected.

For the circuit we are using here, we can find R_{Th} by turning off the independent 15V source:



$$R_{Th} = 2k\Omega + 3k\Omega = 5k\Omega \qquad R_{Th}C = 5k\Omega \cdot 0.2\mu F = 1ms$$

$$\therefore v_c(t>0) = \underset{v_c(t\to\infty)}{\underset{\uparrow}{15V}} + \Big(\underset{v_c(0^-)}{\underset{\uparrow}{0V}} - \underset{v_c(t\to\infty)}{\underset{\uparrow}{15V}}\Big) e^{-t/1ms}$$