HOMEWORK #13 solution prob 4

Ex: Write phasors (as both $Ae^{j\phi}$ and $A\angle\phi$) for each of the following signals:

a) $v(t) = 6\cos(1\text{k}t + 45°)$ V

b) $i(t) = 6\sin(\omega t + 45°)$ mA

c) $i(t) = 5\ \mu\text{F} \cdot \dfrac{d}{dt} 4\cos(1\text{M}t + 45°)$ V

d) $v(t) = 3\ \text{pH} \cdot \dfrac{d}{dt} 2\sin(10t - 30°)$ mA

e) $v(t) = \cos(10t + 60°)$ V $+ 3\sin(10t - 30°)$ V

Sol'n: a) The magnitude of the phasor is the magnitude of the sinusoid, and the phase angle in the exponent of the phasor is the phase shift of the cosine waveform. (The frequency is left out of the phasor, since every signal has the same frequency.)

$$\text{P}[v(t) = 6\cos(1\text{k}t + 45°)\ \text{V}] = 6e^{j45°}\ \text{V}$$

b) The phasor of $\sin(\omega t)$ is $-j$, which adds a minus 90° phase shift to the phasor we would get with a cosine of the same phase shift.

$$\text{P}[i(t) = 6\sin(\omega t + 45°)\ \text{mA}] = 6(-j)e^{j45°}\ \text{mA}$$

or

$$\text{P}[i(t)] = 6e^{-j90°}e^{j45°}\ \text{mA} = 6e^{-45°}\ \text{mA}$$

or

$$\text{P}[i(t)] = 6\angle -45°\ \text{mA}$$

c) When we take a derivative, we multiply the phasor by $j\omega$.

$$\text{P}\left[i(t) = 5\ \mu\text{F} \cdot \frac{d}{dt} 4\cos(1\text{M}t + 45°)\ \text{V}\right] = 5\mu\text{F} \cdot \text{j}\omega \cdot 4e^{j45°}\text{V}$$

or

$$\text{P}\big[i(t)\big] = 5\mu\text{F} \cdot \text{j1Ms}^{-1} \cdot 4e^{j45°}\text{V} = j5 \cdot 4e^{j45°}\mu\text{A} = 20e^{j90°}e^{j45°}\mu\text{A}$$

or

$$P\left[i(t)\right] = 20e^{j135^\circ}\mu\text{A}$$

d) Here, we have multiplication by $j\omega$ for the derivative and $-j$ for sin().

$$P\left[v(t) = 3\text{ pH}\cdot\frac{d}{dt}2\sin(10t-30^\circ)\text{ mA}\right] = 3\text{pH}\cdot j10\text{s}^{-1}\cdot(-j2)e^{-j30^\circ}\text{mA}$$

or, since $-j\cdot j = -1$:

$$P\left[v(t)\right] = 60e^{-j30^\circ}\text{nV}$$

e) We convert the two waveforms to phasors before adding.

$$P[v(t) = \cos(10t+60^\circ)\text{ V} + 3\sin(10t-30^\circ)\text{ V}]$$
$$= e^{j60^\circ} + 3(-j)e^{-j30^\circ}\text{ V}$$

or

$$P\left[v(t)\right] = e^{j60^\circ} + 3e^{-j90^\circ}e^{-j30^\circ}\text{ V} = e^{j60^\circ} + 3e^{-j120^\circ}\text{ V}$$

We could covert each term to rectangular form and sum, but a more efficient approach is to observe that the vectors are colinear. The first phasor is a vector of length 1 at an angle of 60° to the real axis. The second phasor is a vector of length 3 at an angle of –120° to the real axis, which is 180° from the first vector. In other words, the second vector is in the opposite direction from the first vector. Adding the second phasor to the first is like adding a vector of length –3 to a vector of length 1 to get a vector of length –2 in the direction of 60°. Equivalently, we can say that we have a vector of length 2 in the direction of 60° – 180° = –120°.

$$P\left[v(t)\right] = 2\angle -120^\circ\text{ V}$$