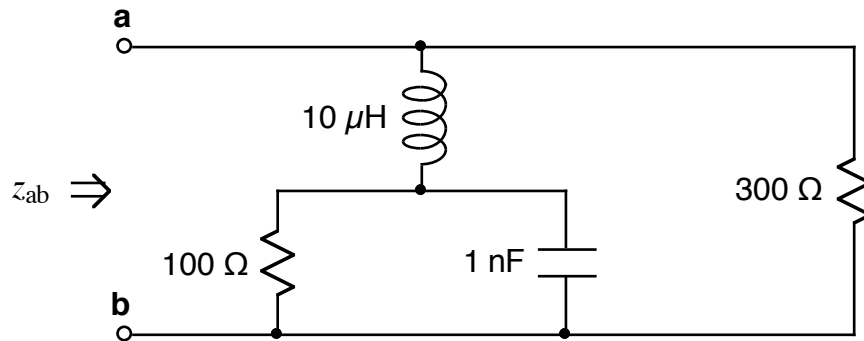


Ex:

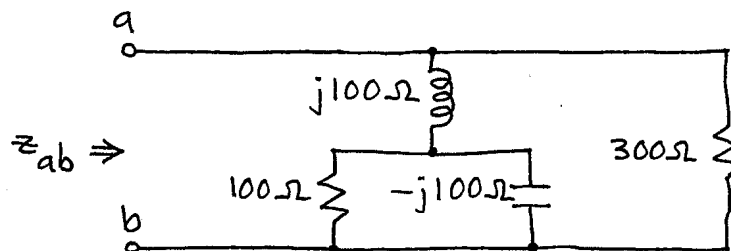


Given $\omega = 10 \text{ M rad/s}$, find z_{ab} .

Sol'n: $z_L = j\omega L = j 10 \text{ Mr/s} \cdot 10 \mu\text{H} = j 100 \Omega$

$$z_C = \frac{-j}{\omega C} = \frac{-j}{10 \text{ Mr/s} \cdot 1 \text{ nF}} = \frac{-j \Omega}{10 \text{ m}} = -j 100 \Omega$$

frequency domain (or s-domain) model:



We first consider the R and C in parallel.

$$100 \Omega \parallel -j 100 \Omega = 100 \Omega \cdot 1 \parallel -j = 100 \Omega \cdot \frac{-j}{1-j}$$

Rationalizing this expression, we have

$$100 \Omega \parallel -j 100 \Omega = 100 \Omega \cdot \frac{-j}{1-j} \cdot \frac{1+j}{1+j}$$

or

$$100\Omega \parallel -j100\Omega = 100\Omega \cdot \frac{1-j}{2} = 100\Omega \cdot \left(\frac{1}{2} - \frac{j}{2}\right)$$

Now we add $z_L = j100\Omega$

$$\begin{aligned} 100\Omega \parallel -j100\Omega + j100\Omega &= 100\Omega \left(\frac{1-j}{2} + j\right) \\ &= 100\Omega \left(\frac{1}{2} + \frac{j}{2}\right) \end{aligned}$$

To find z_{ab} , we use conductance $g_{ab} = \frac{1}{z_{ab}}$:

$$\begin{aligned} g_{ab} &= \frac{1}{100\Omega \parallel -j100\Omega + j100\Omega} + \frac{1}{300\Omega} \\ &= \frac{1}{100\Omega \left(\frac{1}{2} + \frac{j}{2}\right)} + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} \cdot \frac{2}{1+j} \cdot \frac{1-j}{1-j} + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} \cdot \frac{2-j2}{2} + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} (1-j) + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} \left(\frac{4}{3} - j\right) \end{aligned}$$

Now we calculate $\frac{1}{g_{ab}} = z_{ab}$.

$$\begin{aligned}z_{ab} &= \frac{1}{g_{ab}} = 100 \Omega \frac{1}{\frac{4}{3} - j} \\&= 100 \Omega \frac{3}{4 - j3} \\&= 100 \Omega \frac{3}{4 - j3} \cdot \frac{4 + j3}{4 + j3} \\&= 100 \Omega \frac{12 + j9}{4^2 + 3^2} \\&= 100 \Omega \frac{12 + j9}{25} \\&= 4 \cdot (12 + j9) \\z_{ab} &= 48 + j36\end{aligned}$$