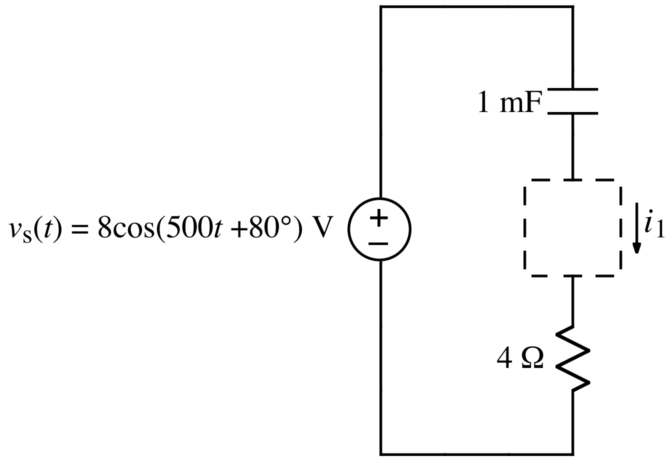
HOMEWORK #16 solution probs 2 and 3

THE UNIVERSITY OF UTAH

Ex:



a) Choose an *R*, an *L*, or a *C* to be placed in the dashed-line box to make

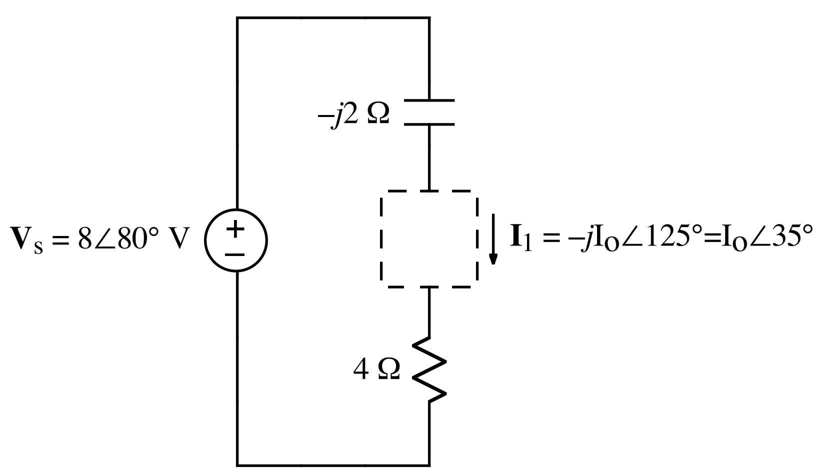
$$i_1(t) = I_o \sin(500t + 125°)$$

where I_o is a positive, (i.e., nonzero and non-negative), real constant. State the value of the component you choose.

b) With your component from part (a) in the circuit, calculate the resulting value of I_o.

SOL'N: a) First, we convert the circuit to the frequency domain, where we have the circuit shown below. Note that for the capacitor, we have $z = 1/j\omega C =$

$$z = \frac{1}{j\omega C} = \frac{1}{j500 \text{ r/s} \cdot 1 \text{ mF}} = -j2\ \Omega$$



Current $\mathbf{I}_1$ is given by the voltage of the source divided by the total impedance in the circuit:

$$\mathbf{I}_1 = I_o\angle 35° = \frac{\mathbf{V}_s}{z_{Tot}} = \frac{8\angle 80°\text{ V}}{4 - j2 + z_{box}}$$

Rearranging gives a formula for z_{box}:

$$4 - j2 + z_{box} = \frac{8\angle 80°\text{ V}}{I_o\angle 35°} = \frac{8\text{ V}}{I_o}\angle 45°$$

We see that the total impedance must have a phase angle of 45°. This is only possible if the total impedance has positive and equal real and imaginary parts. Since the capacitor gives a negative imaginary value, the value of z_{box} must overcome this negative imaginary value. Since z_{box} is the impedance of a single R, L, or C, it can be only purely real (and positive) or purely imaginary. We conclude that z_{box} must be imaginary and positive and yield a total impedance with the real and imaginary parts equal to the real part of the total impedance:

$$4\ \Omega - j2\ \Omega + z_{box} = 4 + j4\ \Omega$$

or

$$z_{box} = j6\ \Omega$$

To obtain this impedance, we use an inductor:

$$z_{box} = j\omega L = j6\ \Omega$$

or

$$\omega L = 6\ \Omega$$

or

$$L = \frac{6\ \Omega}{\omega} = \frac{6\ \Omega}{500} = 12\text{ mH}$$

b) From part (a), we see that $z_{box} = 4 + j4\ \Omega$. We use Ohm's law to find the current, and we take the magnitude of the equation to find I_o:

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{4 + j4\ \Omega} = \frac{8\angle 80°\ \text{V}}{4\sqrt{2}\angle 45°\ \Omega} = 2\sqrt{2}\angle 35°\ \text{A}$$

or

$$\text{I}_o = \left|\frac{\mathbf{V}_s}{4 + j4\ \Omega}\right| = \left|\frac{8\angle 80°\ \text{V}}{4\sqrt{2}\angle 45°\ \Omega}\right| = \left|\frac{8\ \text{V}}{4\sqrt{2}\ \Omega}\right| = 2\sqrt{2}\ \text{A}$$