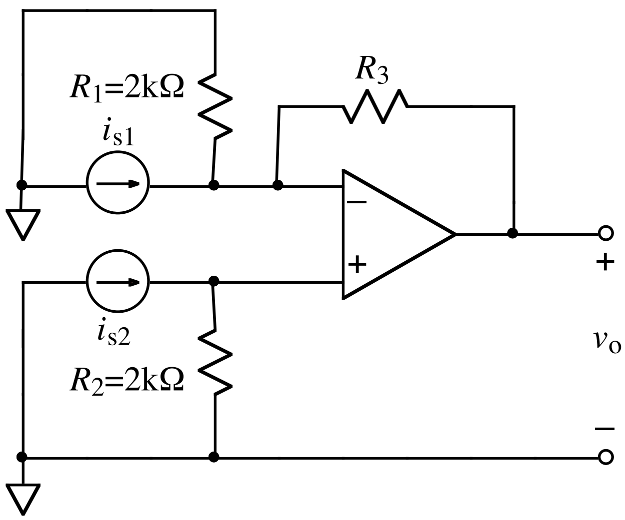
HOMEWORK #17 solution

Ex:



Rail voltages = ±10 V

a) The above circuit operates in linear mode. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_{s1}, i_{s2}, R_1, R_2, and R_3.

b) If $i_{s1} = 0$ μA, find the value of R_3 that will yield an output voltage of $v_o = 1$ V when $i_{s2} = 10$ μA.

c) Derive a symbolic expression for v_o in terms of common mode and differential input currents:

$$i_\Sigma \equiv \frac{i_{s1} + i_{s2}}{2} \qquad \text{and} \qquad i_\Delta \equiv \frac{i_{s1} - i_{s2}}{2}$$

The expression must contain not more than the parameters i_Σ, i_Δ, R_1, R_2, and R_3. Write the expression as i_Σ times a term plus i_Δ times a term. Hint: start by writing i_{s1} and i_{s2} in terms of i_Σ and i_Δ:

$$i_{s1} \equiv i_\Sigma + i_\Delta \qquad \text{and} \qquad i_{s2} \equiv i_\Sigma - i_\Delta$$

d) If $i_\Delta = 0$ and $R_1 = R_2$, write a formula for the current flowing from left to right in R_3 as a function of not more (and possibly less) than the following terms: i_Σ, R_1, R_2, and R_3.

sol'n: a) First, we find v_p (voltage at + input):

$$v_p = i_{s2} R_2$$

Second, we find the current flowing toward the − input from the left, using v_n (voltage at − input) $= v_p$:

$$i_\ell = i_{s1} - \frac{v_n}{R_1} = i_{s1} - \frac{i_{s2} R_2}{R_1}$$

Third, we find the current flowing in the feedback resistor, R_3, from left to right:

$$i_r = \frac{v_n - v_o}{R_3} = \frac{i_{s2} R_2 - v_o}{R_3}$$

Fourth, we set $i_r = i_\ell$ and solve for v_o:

$$i_{s1} - \frac{i_{s2} R_2}{R_1} = \frac{i_{s2} R_2 - v_o}{R_3}$$

or $$v_o = -i_{s1} R_3 + i_{s2} R_2 \left(1 + \frac{R_3}{R_1}\right)$$

b) $$1V = 10\mu A \cdot 2k\Omega \left(1 + \frac{R_3}{2k\Omega}\right) = 20mV \left(1 + \frac{R_3}{2k\Omega}\right)$$

$$\therefore\ 1 + \frac{R_3}{2k\Omega} = 50 \quad \text{or} \quad \frac{R_3}{2k\Omega} = 49$$

or $R_3 = 98\ k\Omega$

c) $$v_o = -(i_\Sigma + i_\Delta)R_3 + (i_\Sigma - i_\Delta)R_2\left(1 + \frac{R_3}{R_1}\right)$$

or

$$v_o = i_\Sigma\left(R_2 + \frac{R_2 R_3}{R_1} - R_3\right)$$

$$- i_\Delta\left(R_2 + \frac{R_2 R_3}{R_1} + R_3\right)$$

d) For $i_\Delta = 0$ and $R_1 = R_2$, we have

$$v_o = i_\Sigma\left(R_2 + \cancel{R_3} - \cancel{R_3}\right)$$

or $v_o = i_\Sigma R_2$

Then $$i_{R3} = \frac{v_n - v_o}{R_3} = \frac{v_n - i_\Sigma R_2}{R_3}$$

But $$v_n = v_p = i_{s2} R_2 = \frac{i_{s1} + i_{s2}}{2} R_2$$

(since $i_{s1} = i_{s2}$) $$= i_\Sigma R_2$$

Then $$i_{R3} = \frac{i_\Sigma R_2 - i_\Sigma R_2}{R_3} = 0\ A$$

Note: When $i_{s1} = i_{s2}$, the current in R_1 and R_2 is the same (since $v_n = v_p$), so there is no current left over to flow in R_3.