HOMEWORK #1 solution

EX: This problem addresses the power and energy consumed by a circuit component.

a) Compute the power as a function of time consumed by a resistor with the following current and voltage waveforms versus time:

$$i(t) = 2 + 3\cos(2\pi t - 45°) \text{ A}$$
$$v(t) = 4 + 6\cos(2\pi t + 45°) \text{ V}$$

b) Find the energy consumed by the component described in (a) in the first second. Note: Convert the 45° to radians as needed.

SOL'N: a) Power is the product of voltage and current.

$$p(t) = i(t) \cdot v(t) = [2 + 3\cos(2\pi t - 45°)][4 + 6\cos(2\pi t + 45°)] \text{ W}$$

or

$$p(t) = 8 + 3(4)\cos(2\pi t - 45°) + (2)6\cos(2\pi t + 45°)$$
$$+ 3(6)\cos(2\pi t - 45°)\cos(2\pi t + 45°) \text{ W}$$

We can simplify the middle cos terms via the trigonometric identity for $\cos(A + B)$, if desired:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B.$$

Now we turn to the second equation:

$$12\cos(2\pi t - 45°) = 12\cos(2\pi t)\cos(-45°) - 12\sin(2\pi t)\sin(-45°)$$

or

$$12\cos(2\pi t - 45°) = 12\cos(2\pi t)\frac{\sqrt{2}}{2} - 12\sin(2\pi t)\frac{-\sqrt{2}}{2}$$

or

$$12\cos(2\pi t - 45°) = 12\frac{\sqrt{2}}{2}[\cos(2\pi t) + \sin(2\pi t)]$$

Similarly, the other middle term becomes

$$12\cos(2\pi t + 45°) = 12\frac{\sqrt{2}}{2}[\cos(2\pi t) - \sin(2\pi t)].$$

Summing the middle terms yields the following result:

$$12\cos(2\pi t - 45°) + 12\cos(2\pi t + 45°) = 12\sqrt{2}\cos(2\pi t)$$

Turning to the last term, we employ an identity for cos(*A*)cos(*B*):

$$\cos A \cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

Apply this to the third term yields the following:

$$18\cos(2\pi t - 45°)\cos(2\pi t + 45°) = 18\left(\frac{1}{2}\right)\cos(2\pi t - 45° + 2\pi t + 45°)$$
$$+18\left(\frac{1}{2}\right)\cos(2\pi t - 45° - (2\pi t + 45°))$$

or

$$18\cos(2\pi t - 45°)\cos(2\pi t + 45°) = 9\cos(4\pi t) + 9\cos(-90°)$$

or, since cos(–90°) = 0,

$$18\cos(2\pi t - 45°)\cos(2\pi t + 45°) = 9\cos(4\pi t).$$

Combining results yields the final answer:

$$p(t) = 8 + 12\sqrt{2}\cos(2\pi t) + 9\cos(4\pi t) \text{ W}$$

b) Energy is the integral of power with respect to time. The product of power and time is energy (or work). The units for energy are Joules.

$$w(t = 1 \text{ s}) = \int_0^1 p(t)dt = \int_0^1 [8 + 12\sqrt{2}\cos(2\pi t) + 9\cos(4\pi t)] \text{ W } dt$$

or

$$w(t = 1 \text{ s}) = \int_0^1 8 \text{ W } dt$$
$$+ \int_0^1 12\sqrt{2}\cos(2\pi t) \text{ W } dt.$$
$$+ \int_0^1 9\cos(4\pi t) \text{ W } dt$$

or

$$w(t=1\text{ s}) = 8t\Big|_0^1 \text{ J} + \frac{12\sqrt{2}\sin(2\pi t)}{2\pi}\Bigg|_0^1 \text{ J} + \frac{9\sin(4\pi t)}{4\pi}\Bigg|_0^1 \text{ J}$$

or

$$w(t=1\text{ s}) = 8(1) - 0\text{ J} + \frac{12\sqrt{2}\sin(2\pi\cdot 1)}{2\pi} - \frac{12\sqrt{2}\sin(0)}{2\pi}\text{ J}. + \frac{9\sin(4\pi\cdot 1)}{4\pi} - \frac{9\sin(0)}{4\pi}\text{ J}$$

or

$$w(t=1\text{ s}) = 8(1) - 0\text{ J} + 0 - 0\text{ J}. + 0 - 0\text{ J}$$

or

$$w(t=1s) = 8\text{ J}.$$

NOTE: The integrals of the cos() terms are zero since the sin() function is evaluated at times where the total angles differ by an integer multiple of 2π and are, therefore, the same.