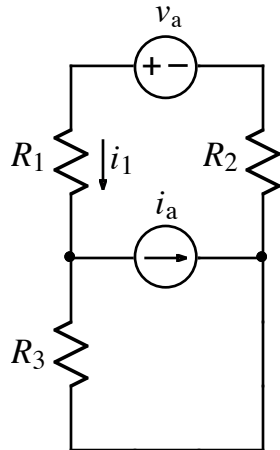


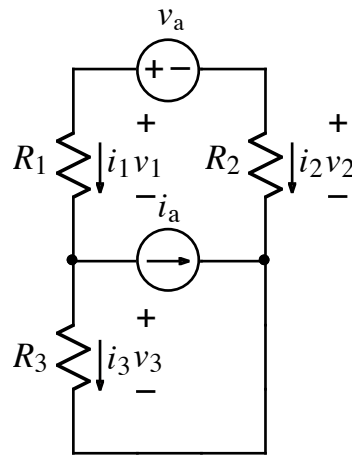
Ex:



Derive an expression for i_1 . The expression must not contain more than the circuit parameters v_a , i_a , R_1 , R_2 , and R_3 .

SOL'N: Using Kirchoff's and Ohm's laws, we write equations for the circuit. The first step is to label the current and voltage for each resistor in accordance with the passive sign convention.

The labeling of voltages and currents may be done in either of two directions, so long as the arrow of the current measurement points toward the minus sign of the voltage measurement. Placing + signs at the left or on top usually works well, however.



By Ohm's law, resistor voltages may be written in terms of resistor currents:

$$v_1 = i_1 R_1$$

$$v_2 = i_2 R_2$$

$$v_3 = i_3 R_3$$

If we use $i \cdot R$ in place of v , we obtain formulas in terms of only currents. This reduces the final number of equations we will generate.

Starting with v -loops, the only v -loop we use is the outer loop, as all other loops would require us to define a voltage for current source i_a .

For the outer loop, traveling clockwise from the lower left yields the following:

$$i_3 R_3 + i_1 R_1 - v_a - i_2 R_2 = 0 \text{ V} \quad (1)$$

Now we consider currents flowing out of nodes, and we consider currents for components in series.

R_1 , v_a , and R_2 are in series and must carry the same physical current. Given that the direction of current measurement for i_2 is opposite to the direction of current measurement for i_1 , we have the following equation:

$$i_2 = -i_1 \quad (2)$$

Summing currents out of the node on the left yields the following equation:

$$-i_1 + i_a + i_3 = 0 \text{ A} \quad (3)$$

Summing currents out of the node on the right yields the same equation multiplied by -1 , (after we substitute $-i_1$ for i_2). Thus, we omit the equation for the right node.

From equation (3), we have an expression for i_3 in terms of i_1 and i_a :

$$i_3 = i_1 - i_a$$

Using this result and equation (2) in equation (1) allows us to write an equation involving i_1 as the only unknown:

$$(i_1 - i_a)R_3 + i_1 R_1 - v_a + i_1 R_2 = 0 \text{ V}$$

or

$$i_1(R_1 + R_2 + R_3) = v_a + i_a R_3$$

or

$$i_1 = \frac{v_a + i_a R_3}{R_1 + R_2 + R_3}$$