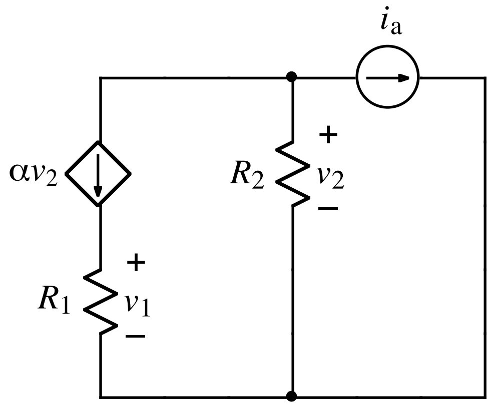
HOMEWORK #4 solution

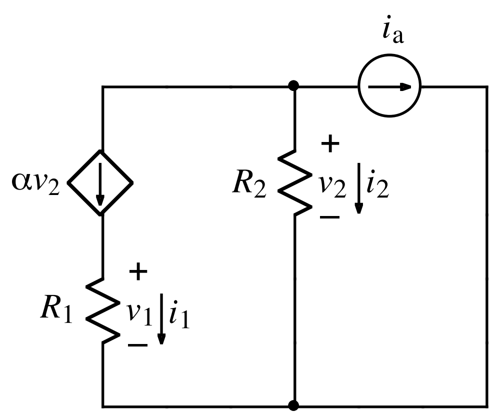
Ex:



a) Derive an expression for v_1. The expression must not contain more than the circuit parameters α, i_a, R_1, and R_2. **Note:** $\alpha > 0$.

b) Make at least one consistency check (other than a units check) on your expression. In other words, choose component values that make it possible to solve the circuit by inspection, and verify that your answer to (a) gives that answer when you plug in those component values. Specify your consistency check by listing a numerical value for every source and resistor.

SOL'N: a) We label voltage and current for R_2 in accordance with the passive sign convention:



We have no v-loops for this circuit since every loop contains a current source.

We observe that the current in R_1 is αv_2 since R_1 is in series with the αv_2 source.

Summing currents out of the top node yields the following equation where we have used Ohm's law in writing i_2:

$$\alpha v_2 + \frac{v_2}{R_2} + i_a = 0 \text{ A}$$

Solving for v_2 yields the following result:

$$v_2\left(\alpha + \frac{1}{R_2}\right) = -i_a$$

or

$$v_2 = \frac{-i_a}{\alpha + \frac{1}{R_2}}$$

NOTE: α changes voltage into current and must have units of $1/\Omega$. Thus, our expression has consistent units.

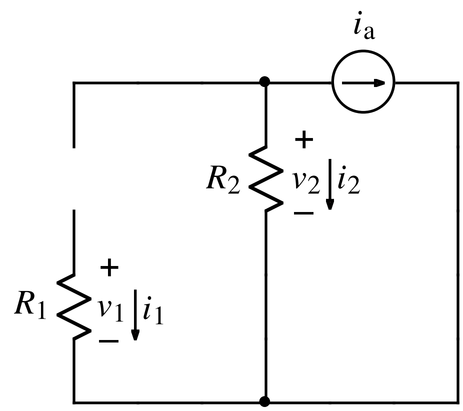
We find v_1 by applying Ohm's law:

$$v_1 = i_1 R_1 = \alpha v_2 R_1 = \frac{-\alpha i_a R_1}{\alpha + \frac{1}{R_2}} = \frac{-\alpha i_a R_1 R_2}{1 + \alpha R_2}$$

b) Various consistency checks are possible. Here, two such checks are presented, although the problem requires only one.

The idea is to choose component values that cause the circuit to become so simple that we can solve it by inspection. We then see if the symbolic answer to (a) gives the same answer when we substitute all the same component values.

Ex 1: Set $\alpha = 0$. Then the dependent current source is off. Zero current corresponds to an open circuit so our circuit is as follows:



Because of the open circuit, i_1 and (by Ohm's law) v_1 are zero.

We compare this result with the answer to (a) when $\alpha = 0$:

$$v_1 = \frac{-\alpha i_a R_1}{\alpha + \frac{1}{R_2}} = \frac{-0 \cdot i_a R_1}{0 + \frac{1}{R_2}} = \frac{0}{\frac{1}{R_2}} = 0 \text{ V}$$

The results agree, so the consistency check is satisfied. $\surd$

Ex 2: Set $\alpha = 1/R_2$. Then the dependent current source produces a current of $i = \alpha v_2 = v_2/R_2 = i_2$ by Ohm's law. This means that the two currents flowing down through the two branches on the left side are the same. These two currents also sum to give $-i_a$ (based on a current summation for the top-center node. It follows that each current is negative one-half times i_a:

$$i_1 = i_2 = -\frac{i_a}{2}$$

By Ohm's law, we find v_1:

$$v_1 = \frac{-i_a R_1}{2}$$

We compare this result with the answer to (a) when $\alpha = 1/R_2$:

$$v_1 = \frac{-\alpha i_a R_1}{\alpha + \frac{1}{R_2}} = \frac{-\frac{1}{R_2} \cdot i_a R_1}{\frac{1}{R_2} + \frac{1}{R_2}} = \frac{-i_a R_1}{2}$$

The results agree, so the consistency check is satisfied. $\surd$