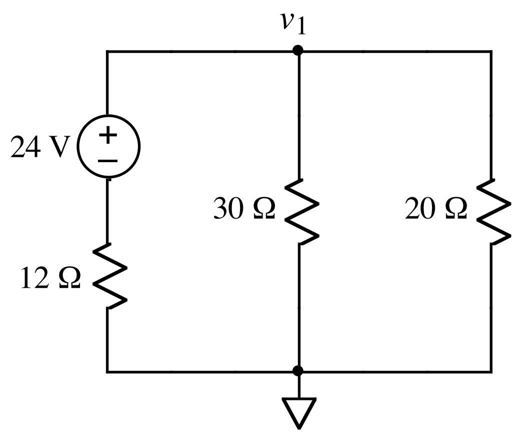
HOMEWORK #6 solution

Ex:



a) Use the node-voltage method to find v_1.

b) Find the equivalent resistance for the 30Ω and 20Ω resistors in parallel. Then use the voltage divider formula to find v_1. Verify that both (a) and (b) have the same answer.

SOL'N: a) We set the sum of currents out of v_1 equal to zero:

$$\frac{v_1 - 24\text{ V}}{12\ \Omega} + \frac{v_1 - 0\text{ V}}{30\ \Omega} + \frac{v_1 - 0\text{ V}}{20\ \Omega} = 0\text{ V}$$

or, if we group terms multiplying v_1 and put constants on the right,

$$v_1\left(\frac{1}{12\ \Omega} + \frac{1}{30\ \Omega} + \frac{1}{20\ \Omega}\right) = \frac{24\text{ V}}{12\ \Omega} = 2\text{ A}$$

or, multiply by the common denominator,

$$60\ \Omega \cdot v_1\left(\frac{1}{12\ \Omega} + \frac{1}{30\ \Omega} + \frac{1}{20\ \Omega}\right) = 60\ \Omega \cdot 2\text{ A}$$

or

$$v_1(5 + 2 + 3) = 120\text{ V}$$

or

$$v_1 = \frac{120\text{ V}}{10} = 12\text{ V}$$

b) First, we calculate the parallel resistance of the 20 Ω and 30 Ω:

$$20\ \Omega \parallel 30\ \Omega = 10\ \Omega \cdot 2 \parallel 3 = 10\ \Omega \frac{2(3)}{2+3} = 10\ \Omega \frac{6}{5} = 12\ \Omega$$

The voltage divider formula gives the following result that agrees with the previous answer:

$$v_1 = 24\ \text{V} \frac{12\ \Omega}{12\ \Omega + 12\ \Omega} = 12\ \text{V}$$

NOTE: For the voltage divider, the 12 Ω on top is from 20 Ω || 30 Ω.