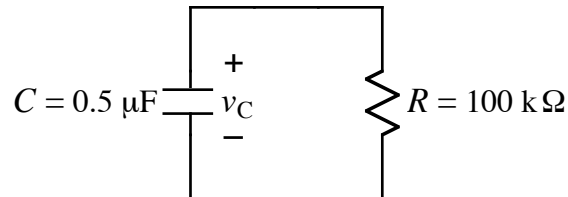
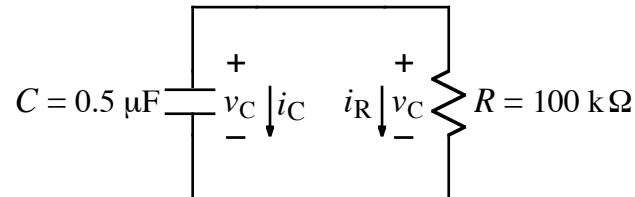


Ex: Find the voltage, v_C , on the capacitor in the circuit below as a function of time if $v_C(t = 0^+) = 6 \text{ V}$.



SOL'N: We observe that voltage v_C appears across both the R and C , as shown below.



Using the defining equation of a capacitor and Ohm's law, we have the following results:

$$i_C = C \frac{dv_C}{dt}$$

and

$$i_R = \frac{v_C}{R}$$

Since the current must be the same everywhere in the loop, and since the currents are measured with opposite polarities, we have that i_C and i_R are equal but opposite.

$$i_C = -i_R$$

or

$$C \frac{dv_C}{dt} = -\frac{v_C}{R}$$

One way to solve this equation is to separate the variables:

$$C \frac{1}{v_C} dv_C = -\frac{1}{R} dt$$

Integrating both sides yields the following result:

$$C \int_{v_C(t=0)}^{v_C(t)} \frac{1}{v_C} dv_C = -\frac{1}{R} \int_0^t dt$$

or

$$C \ln v_C(t) \Big|_{v_C(t=0)}^{v_C(t)} = -\frac{1}{R} t \Big|_0^t$$

or

$$C[\ln v_C(t) - \ln v_C(0)] = -\frac{1}{R} t$$

or

$$\ln \frac{v_C(t)}{v_C(0)} = -\frac{t}{RC}$$

or

$$\frac{v_C(t)}{v_C(0)} = e^{-\frac{t}{RC}}$$

or

$$v_C(t) = v_C(0) e^{-\frac{t}{RC}}$$

Substituting values given in the problem, we have the following answer:

$$v_C(t) = 6V \cdot e^{-\frac{t}{100\text{k}\Omega \cdot 0.5\mu\text{F}}} = 6V \cdot e^{-\frac{t}{50\text{ms}}}$$