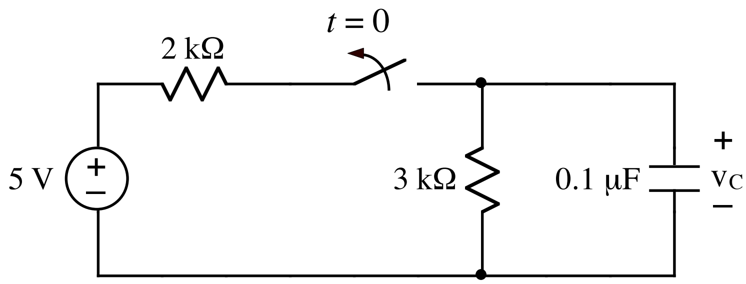
HOMEWORK #11 solution

Ex:



After being closed for a long time, the switch opens at $t = 0$. Find $v_C(t)$ for $t > 0$.
Hint: use a Thevenin equivalent for the voltage source and resistors for $t < 0$.

sol'n: The general sol'n for $v_C(t)$ is

$$v_C(t) = v_C(t\to\infty) + [v_C(t=0^+) - v_C(t\to\infty)]\, e^{-t/\tau}$$

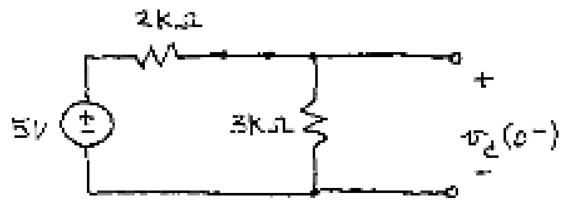
where $\tau = R_{Th}C$.

Note: R_{Th} is the Thevenin equivalent resistance seen looking into the terminals where C is attached.

To find $v_C(0^+)$, we consider $t=0^-$ when the circuit has reached equilibrium. This means currents and voltages are not changing, $\frac{dv_C}{dt} = 0$, and $i_C = C \left.\frac{dv_C}{dt}\right|_{t=0^-} = 0$.

Since $i_C = 0$ at time $t = 0^-$, C acts like an open circuit at $t = 0^-$.

This yields the following circuit model at $t = 0^-$:



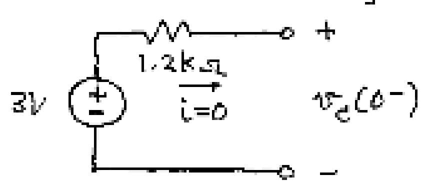
Note that the switch is closed at $t=0^-$.

If we use a Thevenin equivalent for the circuit, we have a voltage divider formed by the 5V, 2kΩ, and 3kΩ.

Thus, $v_{Th} = 5V \cdot \frac{3k\Omega}{2k\Omega + 3k\Omega} = 3V.$

To find R_{Th}, we turn off the 5V source and find that $2k\Omega \| 3k\Omega = 1.2k\Omega$ is the resistance seen looking in from the terminals where we measure $v_c(0^-)$.

$t=0^-$ model using Thevenin equivalent:



Since no current can flow in the circuit owing to the open circuit, the v-drop across the 1.2kΩ resistor is $0 \cdot R = 0$ V.

It follows that $v_c(0^-) = v_{Th} = 3V.$

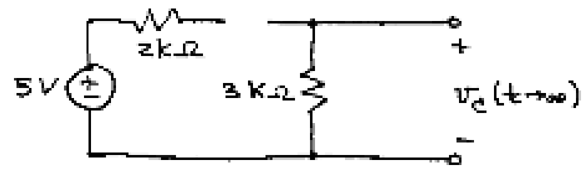
Since v_c is an energy variable, it cannot change instantly. Thus,

$v_c(0^+) = v_c(0^-) = 3V$

To find $v_c(t \to \infty)$, we again employ the assumption that the circuit is in a state where currents and voltages are no longer changing. Thus, $i_c = C\frac{dv_c}{dt} = 0$.

In other words, C acts like an open circuit.

$t \to \infty$ model:



With the switch open, the 5V source and 2kΩ resistor disconnected from the C. Since there is power source in the part of the circuit connected to the C, $v_c(t \to \infty) = 0V$. Note that this satisfies Kirchhoff's voltage law for the loop consisting of the 3kΩ resistor, (with zero current and a zero-volt drop), and the C.

$\therefore\ v_c(t \to \infty) = 0V$

For the time constant, we use the Thevenin equivalent seen looking into the terminals where the C is connected. (Note that we use the circuit for $t > 0$.) Here, the Thevenin equivalent is just 3kΩ R.

$$\tau = R_{Th}C = 3k\Omega \cdot 0.1\mu F$$

or $\quad \tau = 0.3\,ms \quad$ or $\quad 300\,\mu s$

$$\therefore v_C(t) = 0V + [3V - 0V]\,e^{-t/300\mu s}, \quad t \geq 0$$

or $\quad v_C(t) = 3V\,e^{-t/300\mu s}, \quad t \geq 0$