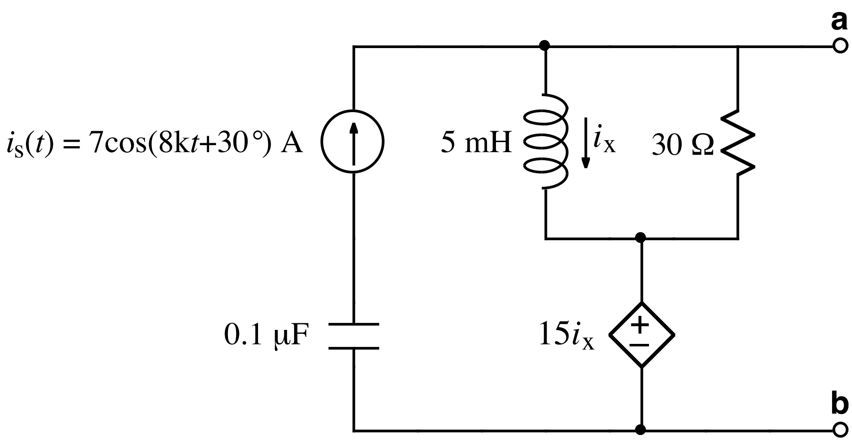
HOMEWORK #16 solution

Ex:



a) Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_S(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b) Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for $\mathbf{V}_{Th}$ and the numerical rectangular form for the impedance value of z_{Th}.

soln: a) We compute phasors and impedances.

$$P[i_s(t)] = P[7\cos(8kt + 30°)A]$$

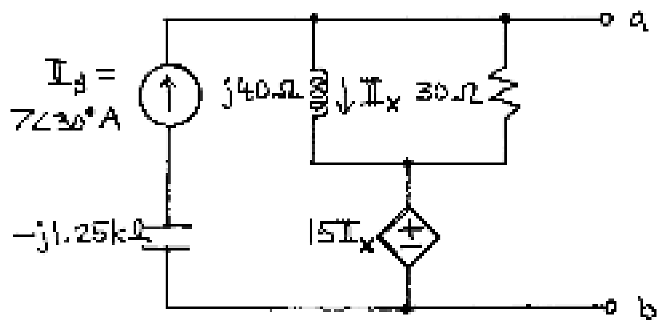
or $\mathbf{I}_s = 7\angle 30°\ A$

$$\frac{-j}{\omega C} = \frac{-j}{8k \cdot 0.1\mu}\ \Omega = -j\,\frac{1k\Omega}{0.8} = -j1.25k\Omega$$

$$j\omega L = j\,8k \cdot 5m\ \Omega = j40\Omega$$

$$\mathbf{I}_x = P[i_x]$$

Frequency- or s-domain model:



b) The −j 1.2kΩ is in series with a current source and may be ignored.

Also, with no load connected across **a** and **b**, (the condition under which we measure V_{Th} across **a** and **b**), the j40Ω and 30Ω form a current divider.

$$\mathbb{I}_x = \mathbb{I}_s \cdot \frac{30\,\Omega}{30\,\Omega + j40\,\Omega}$$

$$= 7\angle 30°\text{A} \cdot \frac{30}{30+j40}$$

$$= 7\angle 30°\text{A} \cdot \frac{30}{50\angle 53.1°}$$

$$= \frac{21}{5}\angle 30° - 53.1°$$

$$= \frac{21}{5}\angle -23.1°$$

To obtain an exact expression for $\mathbb{I}_x$, we may use the following alternative calculation:

$$\mathbb{I}_x = \mathbb{I}_s \cdot \frac{30\,\Omega}{30\,\Omega + j\,40\,\Omega}$$

where $\mathbb{I}_s = 7\angle 30^\circ\ \text{A} = 7\frac{\sqrt{3}}{2} + j\frac{7}{2}$

$$\mathbb{I}_x = 7\cdot\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)\cdot\frac{30\,\Omega}{30\,\Omega + j\,40\,\Omega}\,\frac{30\,\Omega - j\,40\,\Omega}{30\,\Omega - j\,40\,\Omega}\ \text{A}$$

$$= \frac{\frac{21}{2}\left(\sqrt{3} + j\,1\right)\left(3 - j4\right)}{3^2 + 4^2}\ \text{A}$$

$$= \frac{21}{25}\cdot\frac{1}{2}\left(3\sqrt{3} + 4 - j4\sqrt{3} + j\,3\right)\ \text{A}$$

$$\mathbb{I}_x = \frac{21}{50}\left[3\sqrt{3} + 4 + j\left(3 - j4\sqrt{3}\right)\right]\ \text{A}$$

Now we have $V_{Th} = V_{ab}$:

$$V_{Th} = 15\,\mathbb{I}_x + \mathbb{I}_s\ j\,40\,\Omega \,\|\, 30\,\Omega$$

or $V_{Th} = 15\,\mathbb{I}_x + j\,40\,\Omega\,\mathbb{I}_x$

$$= \left(15 + j\,40\,\Omega\right)\frac{21}{50}\left[3\sqrt{3} + 4 + j\left(3 - j4\sqrt{3}\right)\right]\ \text{V}$$

$$= \frac{21}{50}\big(45\sqrt{3} + 60 - 120 + 160\sqrt{3}$$
$$+ j45 - j60\sqrt{3} + j120\sqrt{3} + j\,160\big)\ \text{V}$$

$$V_{Th} = \frac{21}{50}\left[205\sqrt{3} - 60 + j(60\sqrt{3} + 205)\right] V$$

$$V_{Th} = \frac{21}{10}\left[41\sqrt{3} - 12 + j(12\sqrt{3} + 41)\right] \doteq 21\sqrt{73}\angle 46.3^\circ \ V$$

If we opt for an approximate answer, we have the following result:

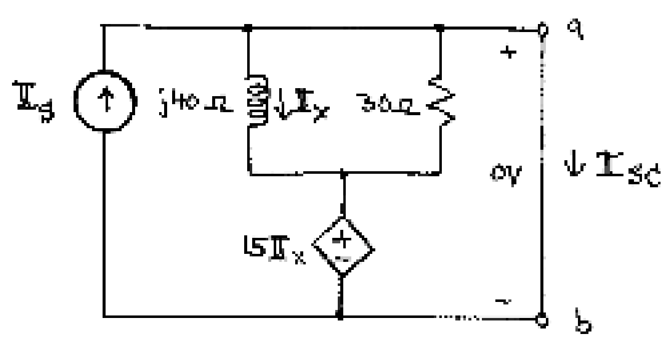
$$V_{Th} = (15 + j40\,\Omega)\,\mathbb{I}_x$$

$$\doteq 5(3+j8)\cdot \frac{21}{5}\angle -23.1^\circ \ V$$

$$= 21\cdot\sqrt{73}\angle 69.4^\circ - 23.1^\circ \ V$$

$$V_{Th} \doteq 21\sqrt{73}\angle 46.3^\circ \ V$$

Now we find Z_{Th} using $Z_{Th} = \dfrac{V_{Th}}{\mathbb{I}_{sc}}$.



We observe that shorting **a** to **b** gives 0V across the components in the middle. We suspect all of $\mathbb{I}_s$ flows in the wire from a to b, meaning $\mathbb{I}_x = 0$ and $15\mathbb{I}_x = 0V$.

We check whether $I_x = 0$ and $15 I_x = 0V$ is plausible. It does because if $15 I_x = 0V$, then we have $0V - 0V = 0V$ across the $j40\Omega$ and 30Ω. Thus, $I_x = \frac{0V}{j40\Omega} = 0$. ✓

So $I_x = 0$ is consistent, and $I_{sc} = I_g$.

$$\therefore \; Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{V_{Th}}{I_g}$$

$$\doteq \frac{21\sqrt{73} \angle 46.3^\circ \text{ V}}{7 \angle 30^\circ \text{ A}}$$

$$\doteq 3\sqrt{73} \angle 46.3^\circ - 30^\circ \; \Omega$$

$$Z_{Th} \doteq 3\sqrt{73} \angle 16.3^\circ \; \Omega$$