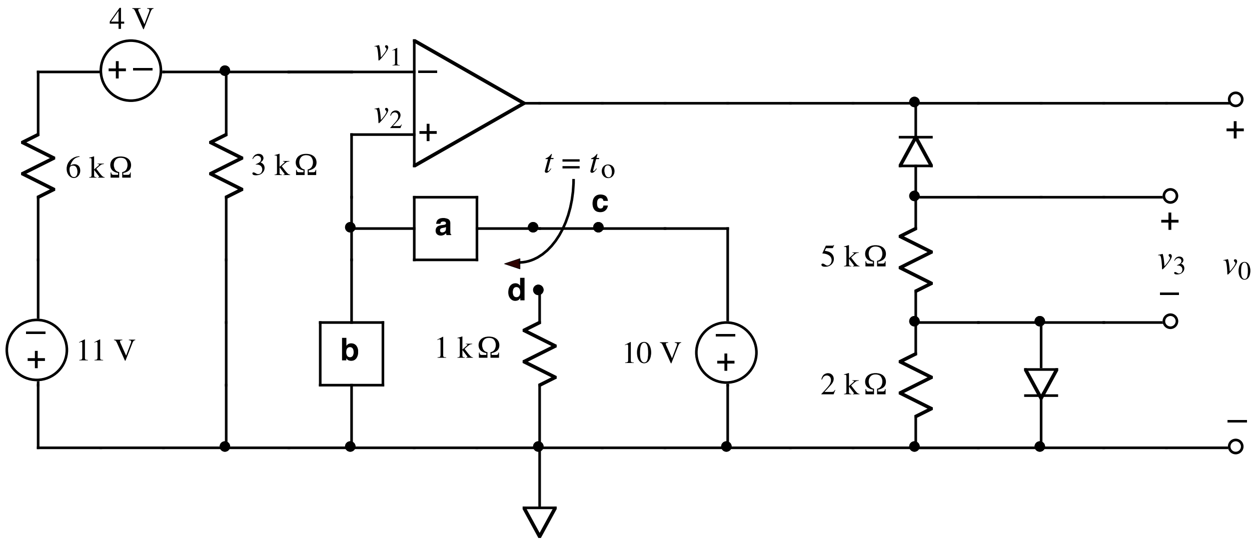
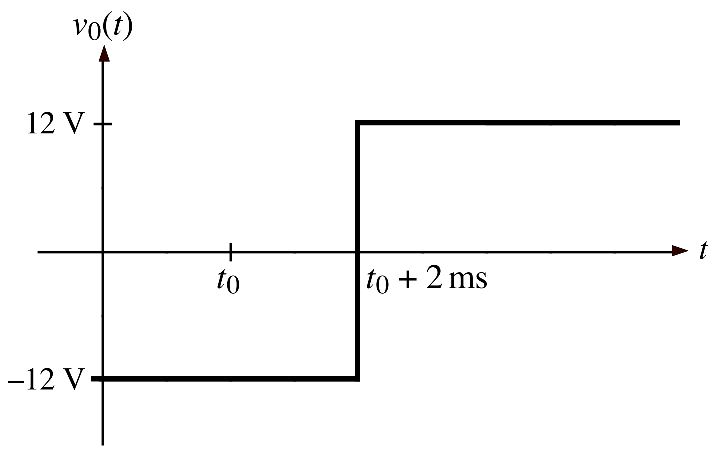
HOMEWORK #18 solution

Ex:



After being in position **c** for a long time, the switch moves from **c** to **d** at $t = t_o$.

Rail voltages = ±12 V



a) Choose either an R or C to go in box **a** and either an R or C to go in box **b** to produce the v_o(t) shown above. (Note that v_o stays high forever after t_o + 2 ms.) Specify which element goes in each box and its value.

b) Sketch v_1(t), showing numerical values appropriately.

c) Sketch v_2(t), showing numerical values appropriately.

d) Sketch v_3(t). Show numerical values for $t < t_o$, for $t_o < t < t_o + 2$ ms, and for $t_o + 2$ ms $< t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

soln: a) For v_o to be low, (i.e., $-12V$), we must have $v_2 < v_1$.

To find v_1, we slide the 4V source through the $6k\Omega$ resistor and find that we have the equivalent of a $-15V$ source and a voltage divider formed by the $3k\Omega$ and $6k\Omega$ resistors.

$$v_1 = -15V \cdot \frac{3k\Omega}{3k\Omega + 6k\Omega} = -5V$$

At $t=0^-$, we must have $v_2 < -5V$.

a = R, b = C [This is possible only if box **a** contains a resistor and box **b** contains a capacitor.] If **a** is an R and **b** is a C, then the C will charge until $v_2 = -10V < v_1$.

When the switch moves from **c** to **d**, the capacitor voltage start charging toward 0V, but it will still be $-10V$ initially. This gives the desired waveform for $v_o(t)$: v_o will go high when $v_2 = v_1 = -5V$.

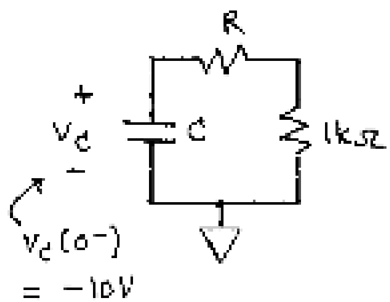
Note: The reasons why other components in boxes **a** and **b** fail to yield the desired $v_o(t)$ are as follows:

a = R and **b** = R cannot give a waveform that changes after a delay. v_o would have to change instantly at $t = t_o$.

a = C and **b** = R would result in C charging until no current flows in R. This means $v_2 = 0V$, or $v_2 > v_1$, causing v_o to be high before $t = t_o$.

a = C and **b** = C would result in an arbitrary voltage at v_2. The total voltage drop across the two C's would be 10V. When the switch changes from **c** to **d**, the capacitors would charge until the total voltage drop across them was 0V. The same current would flow in both C's, causing a voltage change that would be inversely proportional to the C values. The waveform shown for $v_o(t)$ <u>could</u> be produced, but there is a lack of control over the initial value of v_2. This would make the timing of the $v_o(t)$ waveform uncertain. Thus, we reject this solution.

Now we find possible values for R and C. We have the following circuit model for $t > t_0$:



$$v_C(t > t_0) = v_C(t \to \infty) + \left[v_C(t_0^+) - v_C(t \to \infty) \right] e^{-t/\tau}$$

where $v_C(t \to \infty) = 0V$, $v_C(t_0^+) = -10V$, $v_C(t \to \infty) = 0V$

$$v_C(t > t_0) = -10 e^{-t/\tau} \ V \quad (\text{where we take } t_0 = 0)$$

where $\tau = (R + 1k\Omega) C$

We want $v_C(t = 2ms) = v_1 = -5V$

or $-10 e^{-2ms/\tau} \ V = -5V$

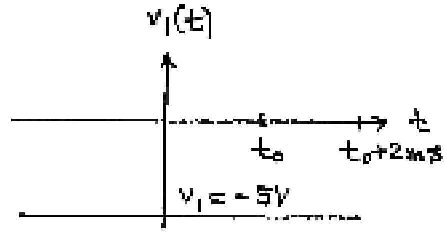
$$e^{-2ms/\tau} = \frac{1}{2}$$

$$-2 \ ms = \tau \ln \frac{1}{2}$$

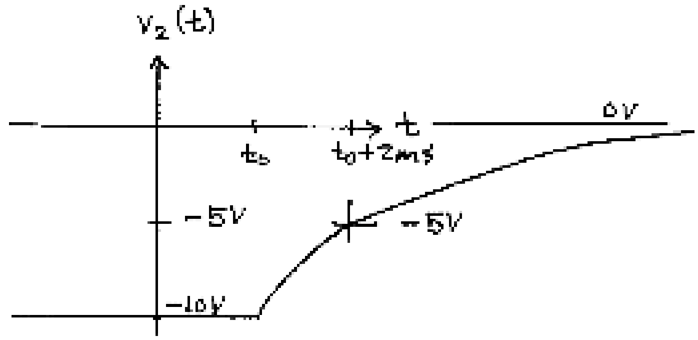
$$\tau = \frac{2 \ ms}{\ln 2} \doteq 2.9 \ ms$$

One sol'n is $R = 1.9 k\Omega$ and $C = 1 \mu F$.
Note: $R = 0 \Omega$ is min R, $C = 2.9 \mu F$ is max C.

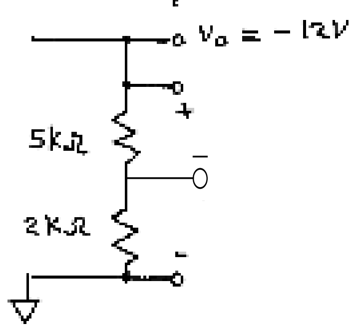
b) $v_1(t) = -5V$ as shown earlier.



c) $v_2 = v_c(t>0) = -10V\,e^{-t/2.9ms}$ from (a)



d) When v_o is low, the top diode will act like a wire and the bottom diode will act like an open circuit.



We have a voltage divider: $v_3 = -12\text{ V}\cdot\frac{5\text{ k}\Omega}{2\text{ k}\Omega + 5\text{ k}\Omega} = -\frac{60}{7}\text{ V}.$

When v_0 is high, the top diode will act like an open circuit, leaving the bottom part of the circuit disconnected from v_0, (or any other power source).

Thus $v_3 = 0V$ when v_0 is high.