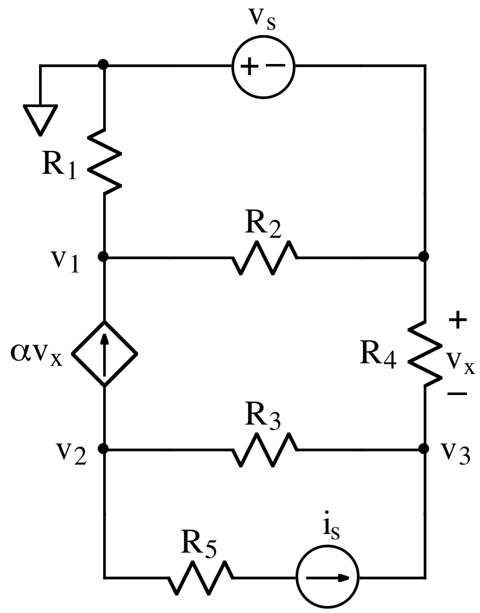
HOMEWORK #8 solution

Ex:



For the circuit shown, write three independent equations for the node-voltages, v_1, v_2, and v_3. The quantity v_x must not appear in the equations.
Make at least one consistency check (other than a units check) on your equations. In other words, choose component values that make the values of v_1, v_2, and v_3 obvious, and verify that your equations give these values. Specify your consistency check by listing a numerical value for every source and resistor.

sol'n: 1) First, we write v_x in terms of node voltages. Because it is connected to reference by only voltage source v_s, the voltage at the upper node on the right side is $-v_s$.

$$v_x = -v_s - v_3$$

Note: we subtract the voltage for the node next to the minus sign of the v_x measurement.

Second, we check to see if node v_1 is a supernode. In other words, we check to see if v_1 is connected to another node by only a voltage source. Since this is not the case, we write a standard node voltage eq'n for node v_1.

$$\frac{v_1 - 0V}{R_1} + \frac{v_1 - -v_s}{R_2} - \alpha(\underbrace{-v_s - v_3}_{v_x}) = 0\,A \quad (1)$$

We also write standard current sums for nodes v_2 and v_3, since they are not supernodes.

$$\alpha(-v_s - v_3) + \frac{v_2 - v_3}{R_3} + i_s = 0\,A \qquad (2)$$

$$\frac{v_3 - -v_s}{R_4} + \frac{v_3 - v_2}{R_3} - i_s = 0\,A \qquad (3)$$

2) For the consistency check, we choose component and source values that make the values of node voltages obvious. Then we verify that the eq'ns from above give the expected answers.

Many checks are possible. One example is shown here.

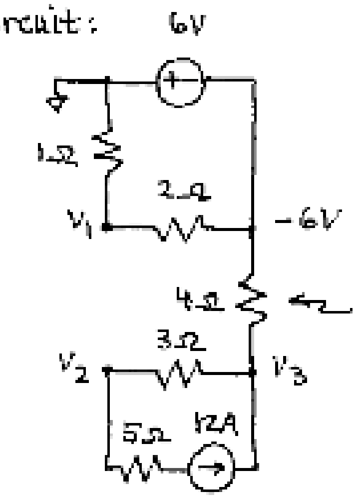
Let $\alpha = 0$ (so dependent src disappears)

$v_s = 6V$

$i_s = 12A$

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$, $R_5 = 5\Omega$.

Circuit:



← No current can flow thru 4Ω resistor since there is no other path by which current can flow out to maintain a net zero charge, as required by Kirchhoff's law. (Consider a bubble around the bottom of the circuit. The current out of the bubble must be zero.)

Since no current flows in the 4Ω resistor, the voltage drop across it is zero. Thus, $v_3 = -6V$.

Also, since no current flows in the 4Ω resistor, we have a voltage divider formed by the 1Ω and 2Ω:

$$\therefore\ v_1 = -6V \cdot 1\Omega / (1\Omega + 2\Omega) = -2V$$

Finally, 12A from the current source flows thru the 3Ω and 5Ω resistors.

Thus, $v_2 = v_3 - 12A \cdot 3\Omega$

$v_2 = -6V - 36V$

$v_2 = -42V$

Now we plug values into the complete node-voltage eq'ns:

(1) $\frac{-2 - 0V}{1\Omega} + \frac{-2 - -6V}{2\Omega} - 0(-6 - -6V)$

$= -2A + 2A - 0$

$= 0A$ ✓ eq'n is satisfied

(2) $0(-6 - -6V) + \frac{-42 - -6V}{3\Omega} + 12A$

$= 0 - 12A + 12A$

$= 0A$ ✓ eq'n is satisfied

(3) $\frac{-6 - -6V}{4\Omega} + \frac{-6V - -42V}{3\Omega} - 12A$

$= -0A + 12A - 12A$

$= 0A$ ✓ eq'n is satisfied