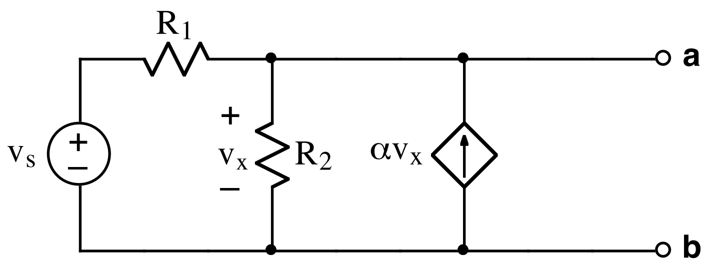
HOMEWORK #8 solution

Ex:



Find the Thevenin equivalent circuit at terminals a-b. v_X must not appear in your solution. **Hint:** use the node-voltage method.

sol'n: $v_{Th} = v_{a,b}$ open circuit

We may use any method we desire to solve for v_{Th}. Here, we use the node-voltage method.

$$v_X = v_{Th} - 0\,V = v_{Th} \quad \text{for dependent src}$$

Current sum for v_{Th} node (dashed bubble):

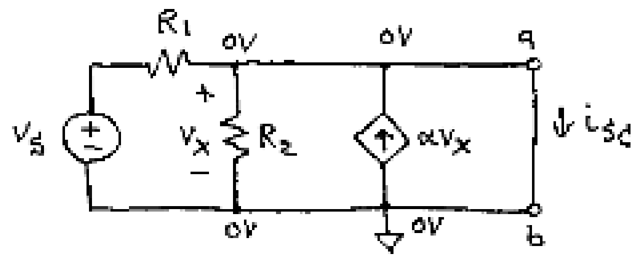
$$\frac{v_{Th} - v_s}{R_1} + \frac{v_{Th}}{R_2} - \alpha v_{Th} = 0 \text{ A}$$

$$\text{or} \quad v_{Th}\left(\frac{1}{R_1} + \frac{1}{R_2} - \alpha\right) = \frac{v_s}{R_1}$$

$$\text{or} \quad v_{Th} = \frac{v_s}{R_1} \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} - \alpha}$$

$$v_{Th} = \frac{v_s}{1 + \frac{R_1}{R_2} - \alpha R_1} \quad \text{or} \quad \frac{v_s}{R_1} \cdot R_1 \| R_2 \| -\frac{1}{\alpha}$$

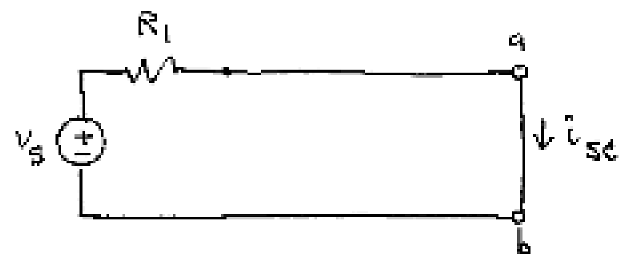
To find R_{Th}, we short a to b and find the short-circuit current, i_{sc}, that flows from a to b.



The wire from a to b means the top rail is at 0V. Thus, $v_x = 0V - 0V = 0V$

Also, it follows that $\alpha v_x = 0$ so the dependent current source effectively disappears.

Furthermore, R_2 is bypassed by the short, meaning no current flows in R_2. Our equivalent circuit model is as follows:



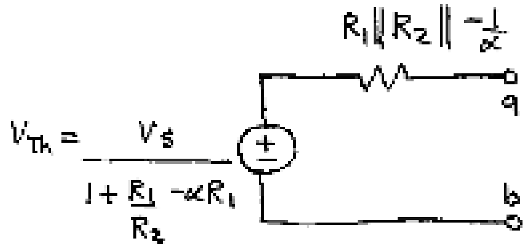
We have $i_{sc} = \dfrac{v_s}{R_1}$.

Now we use the formula $R_{Th} = \dfrac{v_{Th}}{i_{sc}}$.

$$R_{Th} = \frac{\dfrac{V_s}{1 + \dfrac{R_1}{R_2} - \alpha R_1}}{\dfrac{V_s}{R_1}} = \frac{R_1}{1 + \dfrac{R_1}{R_2} - \alpha R_1}$$

$$= \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} - \alpha} \quad \text{or} \quad R_1 \| R_2 \| -\frac{1}{\alpha}$$

Thevenin equivalent circuit:



Note: In this circuit, we have voltage $-V_x$ across the dependent source and can use Ohm's law to find a resistance equivalent to the dependent source:

$$R_{eq} = \frac{-V_x}{\alpha V_x} = -\frac{1}{\alpha}$$

We can use R_{eq} in place of the dependent src, leading to a simpler solution for V_{Th} from a v-divider and for R_{Th} from looking into the a, b terminals with v_s off.