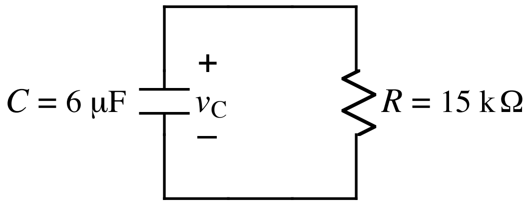
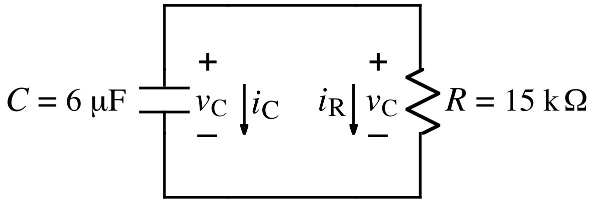
HOMEWORK #9 solution

EX: Find the voltage, v_C, on the capacitor in the circuit below as a function of time if $v_C(t = 0) = 2.4$ V.



SOL'N: We observe that voltage v_C appears across both the R and C, as shown below.



Using the defining equation of a capacitor and Ohm's law, we have the following results:

$$i_C = C\frac{dv_C}{dt}$$

and

$$i_R = \frac{v_C}{R}$$

Since the current must be the same everywhere in the loop, and since the currents are measured with opposite polarities, we have that i_C and i_R are equal but opposite.

$$i_C = -i_R$$

or

$$C\frac{dv_C}{dt} = -\frac{v_C}{R}$$

One way to solve this equation is to separate the variables:

$$C\frac{1}{v_C}dv_C = -\frac{1}{R}dt$$

Integrating both sides yields the following result:

$$C\int_{v_C(t=0)}^{v_C(t)} \frac{1}{v_C}\,dv_C = -\frac{1}{R}\int_0^t dt$$

or

$$C \ln v_C(t)\Big|_{v_C(t=0)}^{v_C(t)} = -\frac{1}{R}t\Big|_0^t$$

or

$$C\left[\ln v_C(t) - \ln v_C(0)\right] = -\frac{1}{R}t$$

or

$$\ln\frac{v_C(t)}{v_C(0)} = -\frac{t}{RC}$$

or

$$\frac{v_C(t)}{v_C(0)} = e^{-\frac{t}{RC}}$$

or

$$v_C(t) = v_C(0)e^{-\frac{t}{RC}}$$

Substituting values given in the problem, we have the following answer:

$$v_C(t) = 2.4\text{V}\cdot e^{-\frac{t}{15\text{k}\Omega\cdot 6\mu\text{F}}} = 2.4\text{V}\cdot e^{-\frac{t}{90\text{ms}}}$$