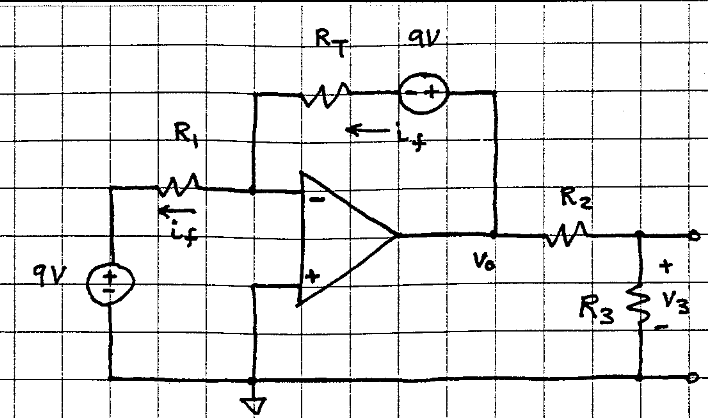
PRACTICE FINAL EXAM
SOLUTION Probs 1 and 2

1.



Rail voltages ±8V

$$R_T = R_o e^{2000\left(\frac{1}{T} - \frac{1}{300}\right)} \qquad R_o = 12k\Omega \quad T \text{ in } {}^\circ K$$

a) Choose components that give $V_3 = 0V$ when $T = 273\,{}^\circ K$
$V_3 = 1V$ $T = 373\,{}^\circ K$

sol'n: First we find V_3 as a function of R_T. Assume linear mode and ideal op-amp:

1) $V_p = 0V$ Always find V_+ (i.e. V_p) first
(or V_+ if you prefer to call it that)

2) $V_n = V_p = 0V$ $V_- = V_+$ (i.e. $V_n = V_p$) for linear mode

3) i_f thru R_T and 9V in feedback $= i_f$ thru R_1 and 9V on left
since i_n (or i_-) $= 0A$ (no input current).

Call op-amp output V_o.

$$i_f = \frac{V_o - 9V - V_n}{R_T} = \frac{V_o - 9V}{R_T} \quad \text{for feedback}$$

$$i_f = \frac{V_n - 9V}{R_1} = -\frac{9V}{R_1} \quad \text{for input } V_n \text{ from left}$$

$$\therefore \frac{V_o - 9V}{R_T} = \frac{-9V}{R_1} \quad \text{or} \quad \frac{V_o}{R_T} = 9V\left(\frac{1}{R_T} - \frac{1}{R_1}\right)$$

$$V_o = 9V\left(1 - \frac{R_T}{R_1}\right)$$

1. a) (cont) $v_o = 0V$ when $R_T = R_1$

$$v_3 = v_o \cdot \frac{R_3}{R_2 + R_3} = 0 \quad \text{when } v_o = 0V$$

$$\text{So } R_1 = R_T(T = 273°K) = 12k\, e^{2000\left(\frac{1}{273} - \frac{1}{300}\right)} = 23.2k$$

for $T = 373°K$: $$v_o = 9V\left[1 - \frac{R_T(373°K)}{23.2k}\right] = 9V\left[\frac{1 - 12k\,e^{2000\left(\frac{1}{373} - \frac{1}{300}\right)}}{23.2k}\right]$$

Note $R_T(373°K) = 3.25k$

$v_o = 7.74\,V$ not quite saturated (but close!)

For $v_3 = 1V$, we need $\frac{R_3}{R_2 + R_3} = \frac{1}{7.74}$

or $\frac{R_2 + R_3}{R_3} = 1 + \frac{R_2}{R_3} = 7.74$

$\therefore \frac{R_2}{R_3} = 6.74$

Use $R_2 = 6.74\,k\Omega$ (or 6.8k if 10% R's required)
$R_3 = 1\,k\Omega$

In practice we would use a $10k\Omega$ potentiometer for R_2, or 510Ω in series with $1k\Omega$ pot for R_1.

Note: Any R_2 and R_3 in the proper ratio is mathematically correct, but the values I chose limit the op-amp output current to $i_f + \frac{7.74V}{R_2 + R_3} = i_f + \frac{7.74V}{7.74k} = i_f + 1mA$

i_f is on the order of $\frac{7.74V - 9V}{3.25k} = -0.39\,mA$

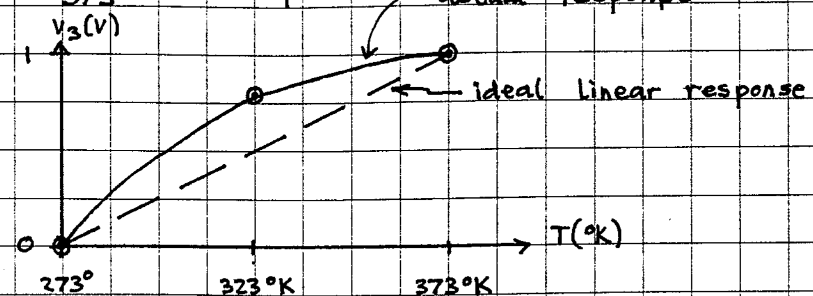
So we have $i_f \lessapprox 2\,mA < 10mA$ limit typical for max op-amp output current.

1. b) Calculate R_T ($T = 323\,°K$). Make rough sketch of V_3 vs T.
On same axes, plot ideal linear response. Comment on results.

sol'n: $R_T(T = 323\,°K) = 12k\, e^{2000\left(\frac{1}{323} - \frac{1}{300}\right)} = 7.46\,k$

$$V_3 = \frac{9V}{7.74}\left[1 - \frac{7.46k}{23.2k}\right] = 0.79\,V$$

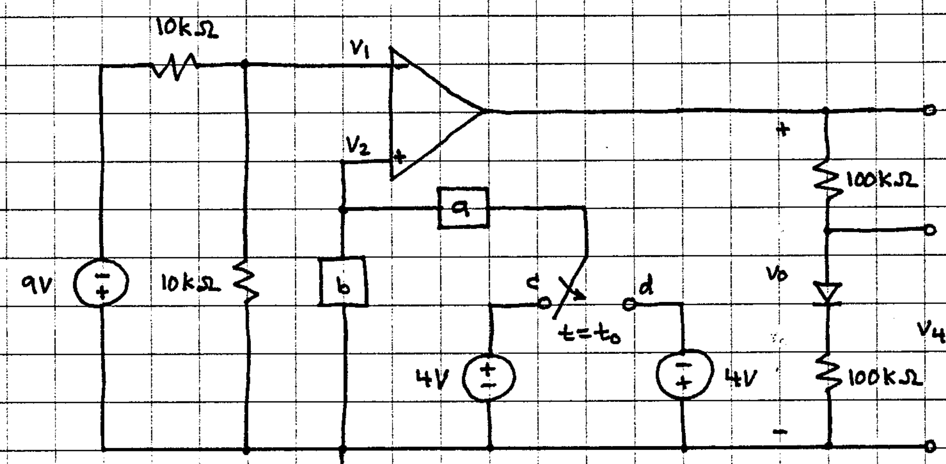
T (°K)	V_3 (V)
273°K	0
323	0.79
373	1



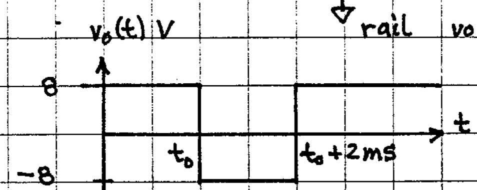
Comment: *The actual response is quite nonlinear.*
It is off by 0.79 actual − 0.5 desired = 0.29
at T = 323 °K. This is an error of $\frac{0.29}{0.50} \approx 60\%$.

This circuit would not be a good thermometer.

2.



rail voltages = ±8V

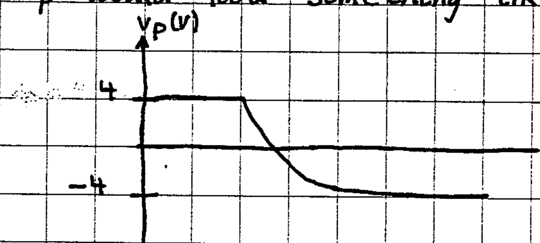


switch moves from c to d at t_0.

2.a) Choose R or C for 'a' and 'b' to produce $v_o(t)$ as shown.

soln: $V_n = -9V \cdot \frac{10k\Omega}{10k\Omega + 10k\Omega} = -4.5V$

To change the output to −8V for the op-amp, we have to have $v_p < v_n = -4.5V$. If we put a C in 'b' and an R in 'a', then the C would charge up to 4V and down to −4V when the switch goes from 'c' to 'd'. The voltage at v_p would look something like this:



We would never have $v_p < -4.5V$. So this is not the right solution.

The other choice is to put a C in 'a' and an R in 'b'. When the C is charged, we will have +4V across the C and no current will be flowing at t_o^-. Since there is no current there will be no V drop across R, and $v_p = 0V$ for $t = t_o^-$.

When we switch from 'c' to 'd', we will start with 4V across C from 'c' to 'd'. Added to the −4V source connected to 'd', we will have $v_p = -8V$ at $t = t_o^+$. This will give $v_p < v_n$ and cause the op-amp output to go negative, as desired.

Starting at t_o^+, the C will start charging toward −4V across C. When the C starts charging, the current thru (and V drop across) R will start declining exponentially to zero from an initial maximum of $8V/R$ (and V drop of 8V).

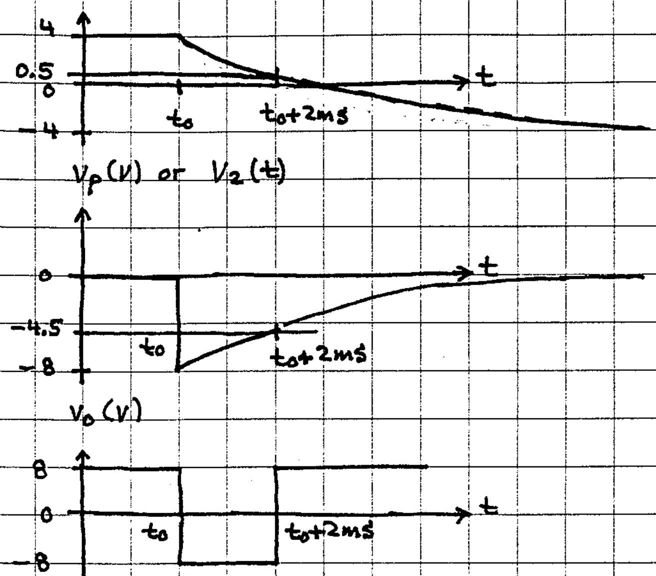
2. a) (cont) When v_p reaches the value of $v_n = -4.5V$, the output of the op-amp will again go high.

Thus, we get the behavior we are seeking.

$\therefore$ Put a C in 'a' and R in 'b'.

The waveforms:

$v_c(V) \leftarrow V$ across C with − on left + on right:



The general sol'n for $v_p(t)$ is clearly $v_p(t') = -8V e^{-t'/RC}$, where $t' = 0$ when $t = t_0$, (i.e. shift time origin).

We want $v_p(t' = 2ms) = -4.5V$.

$$-4.5V = -8\, e^{-2ms/RC}$$

$$\ln \frac{4.5}{8} = -2ms/RC$$

$$RC = \frac{-2ms}{\ln \frac{4.5}{8}} = 3.5\ ms$$

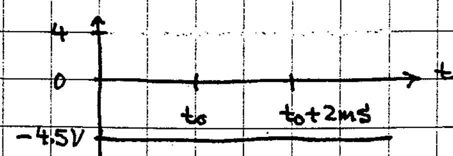
$C = 1\mu F$ $\quad R = 3.5k$ (or 3.6k 10% value)

Other answers with $RC = 3.5ms$ acceptable if R not too small and C not too large.

2\. b) Sketch $v_2(t)$. sol'n: see middle sketch on previous page.

c) Sketch $v_1(t)$.

sol'n: $V_1(t) = -9V \cdot \dfrac{10k\,\Omega}{10k+10k\,\Omega} = -4.5V$ all the time

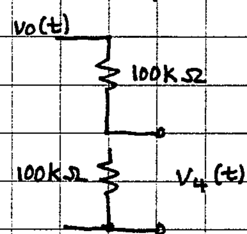


d) Sketch $V_4(t)$. Use ideal model of diode.

sol'n: When $v_0(t) > 0V$ the diode is forward biased and we have V-divider: (diode acts like wire)

$$V_4(t) = V_0(t) \cdot \frac{100k\Omega}{100k\Omega + 100k\Omega} = \frac{V_0(t)}{2} \qquad V_0(t) > 0V$$

When $v_0(t) < 0V$, the diode is reverse biased and acts like an open circuit:



Since there is no load, no current flows in the 100kΩ in series with $v_0(t)$.

Thus, $V_4(t) = V_0(t) \qquad v_0(t) < 0.$