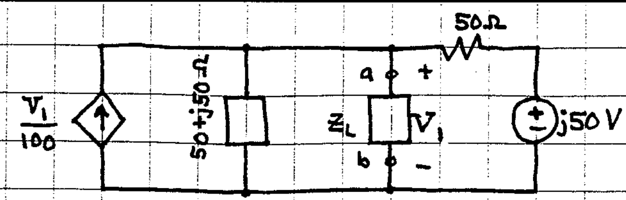
PRACTICE FINAL EXAM
SOLUTION Prob 3

3.



a) Choose Z_L that will absorb max ave power. Hint $Z_L = Z^*_{Thev}$.

sol'n: Take Thevenin equivalent of circuit with respect to terminals a–b (without Z_L).

V_{Th} = open circuit V_{a-b} or V_1.

Use Node-V method to find V_1:

$$-\frac{V_1}{100} + \frac{V_1}{50+j50\,\Omega} + \frac{V_1 - j50V}{50\,\Omega} = 0A$$

$$V_1\left(-\frac{1}{100} + \frac{1}{50+j50\,\Omega} + \frac{1}{50\,\Omega}\right) = \frac{j50V}{50\,\Omega}$$

$$V_1\left(-\frac{(50+j50\,\Omega)}{100} + 1 + \frac{50+j50\,\Omega}{50\,\Omega}\right) = j\frac{50V}{50\,\Omega}(50+j50\,\Omega)$$

$$V_1\left(-\frac{1}{2} - j\frac{1}{2} + 1 + 1 + j\right) = j(50+j50)\ V$$

$$V_1\left(\frac{3}{2} + j\frac{1}{2}\right) = -50 + j50\ V$$

$$V_1(3+j) = -100 + j100\ V$$

$$V_1 = \frac{-100+j100}{3+j} = \frac{-100+j100}{10}(3-j)\ V$$

$$V_1 = (-10+j10)(3-j) = -30+10+j(30+10)\ V$$

$$V_{Th} = V_1 = -20 + j40\ V$$

To find Z_{Th} we connect a 1V source at a & b and turn the independent j50V source to 0V.

We must keep the dependent source, however.

3. a) (cont.) $\quad Z_{Th} = \dfrac{1V}{\mathbb{I}_a}$ where $\mathbb{I}_a$ flows into 'a' terminal with $V_1 = 1\angle 0° \, V$.

$$\mathbb{I}_a = \frac{-V_1}{100} + \frac{V_1}{50+j50\,\Omega} + \frac{V_1}{50\,\Omega}$$ (remember, j50V source is now 0V, or wire)

but $V_1 = 1V$ so $Z_{Th} = \dfrac{1V}{-\dfrac{1V}{100} + \dfrac{1V}{50+j50\,\Omega} + \dfrac{1V}{50\,\Omega}}$

$$Z_{Th} = \frac{1}{-\dfrac{1}{100} + \dfrac{1}{50+j50} + \dfrac{1}{50}}\ \Omega$$

$$= \frac{50+j50}{-\dfrac{50-j50}{100} + 1 + \dfrac{50+j50}{50}}$$

$$= \frac{50+j50}{-\frac{1}{2} - j\frac{1}{2} + 1 + 1 + j} = 50\,\frac{1+j}{\frac{3}{2} + j\frac{1}{2}}$$

$$= 100\,\frac{1+j}{3+j}\,\frac{3-j}{3-j} = \frac{100}{10}\left[3+1+j(3-1)\right]$$

$$= 10\,(4+j2)$$

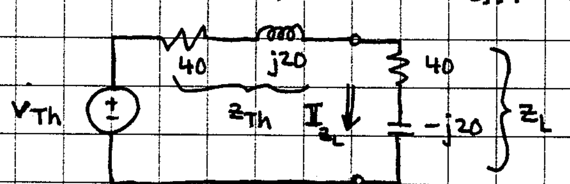
$$Z_{Th} = 40 + j20\ \Omega$$

For max ave power in Z_L we use $Z_L = Z_{Th}^*$.

$$\therefore\ Z_L = 40 - j20\ \Omega$$

b) Calculate max ave power absorbed by Z_L.

sol'n: $\quad P \equiv$ ave power $= |\mathbb{I}_{eff}|^2 \cdot R_L = \left|\dfrac{\mathbb{I}_{Z_L}}{\sqrt{2}}\right|^2 \cdot R_L = \left|\mathbb{I}_{Z_L}\right|^2 \dfrac{}{2} \cdot R_L$



$$\mathbb{I}_{Z_L} = \frac{V_{Th}}{40+j20+40-j20\,\Omega} = \frac{V_{Th}}{80\,\Omega}$$

$$\therefore\ P = \frac{1}{2}\frac{|V_{Th}|^2}{(80\,\Omega)^2} \cdot 40\,\Omega = \frac{1}{2}\frac{(-20)^2 + 40^2}{80^2} \cdot 40\ W = \frac{1}{\cancel{2}} \cdot \frac{\overset{10}{\cancel{20}}}{\underset{16}{\cancel{64}}} \cdot \overset{10}{\cancel{40}} = 6.25\ W$$