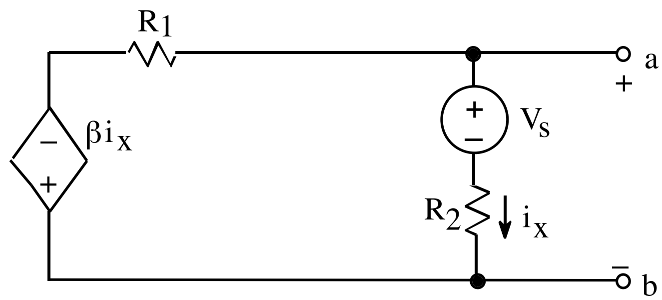
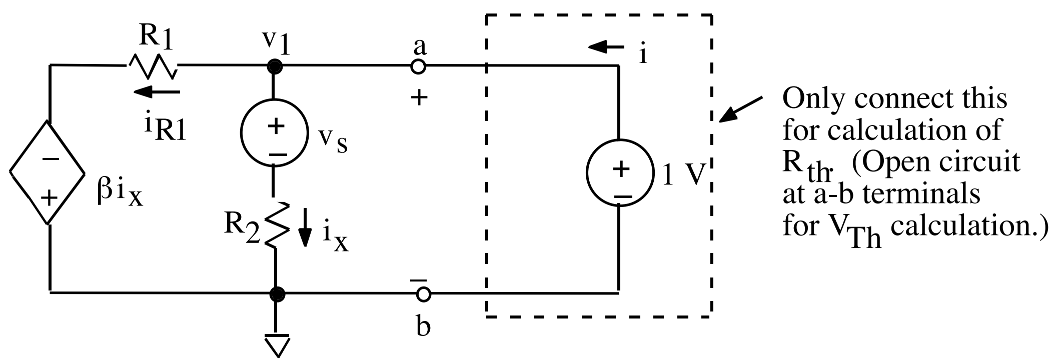
2.



Find the Thevenin's equivalent circuit at terminals a-b. Hint: Use node voltage method to find V_{Th}.

Solution:



First, we find V_{Th} by node-V method (with nothing connected at a-b). We have one node, v_1, to write an equation for: ($V_{Th} = v_1$ no load)

$$\frac{v_1 + \beta i_x}{R_1} + \frac{v_1 - v_s}{R_2} = 0 \text{ A}$$

We have $i_x = \dfrac{v_1 - v_s}{R_2}$. Substitute this into our equation

$$\frac{v_1 + \beta\left(\dfrac{v_1 - v_s}{R_2}\right)}{R_1} + \frac{v_1 - v_s}{R_2} = 0 \text{ A}$$

or

$$v_1\left(\frac{1}{R_1}+\frac{\beta}{R_1R_2}+\frac{1}{R_2}\right)=v_s\left(\frac{1}{R_2}+\frac{\beta}{R_1R_2}\right)$$

Multiply both sides by R_1R_2:

$$v_1\left(R_2+\beta+R_1\right)=v_s\left(R_1+\beta\right)$$

$$\boxed{V_{Th}=v_1=v_s\frac{R_1+\beta}{R_2+\beta+R_1}}$$

To find R_{Th}, we have two possible approaches:

1. Find current in wire shorting a to b. Then use $R_{Th}=\dfrac{V_{Th}}{i_{ab}}$.
2. Connect V source to ab with independent source $v_s = 0$.

   Then use $R_{Th}=\dfrac{v}{i}$ where i = current flowing into "a" terminal.

We'll use the second approach. Set $v_s = 0$V. Connect 1 V source to a-b. Then

$$i_x=\frac{1\,V}{R_2},\qquad i_{R_1}=\frac{1\,V+\beta i_x}{R_1}=\frac{1\,V+\beta\cdot\frac{1\,V}{R_2}}{R_1}=1\,V\left(\frac{1}{R_1}+\frac{\beta}{R_1R_2}\right)$$

$$i=i_x+i_{R_1}=1\,V\left(\frac{1}{R_2}+\frac{1}{R_1}+\frac{\beta}{R_1R_2}\right)=1\,V\left(\frac{R_1+R_2+\beta}{R_1R_2}\right)$$

$$\boxed{R_{Th}=\frac{1\,V}{i}=\frac{1}{\frac{1}{R_2}+\frac{1}{R_1}+\frac{\beta}{R_1R_2}}=R_1\|R_2\|\frac{R_1R_2}{\beta}=\frac{R_1R_2}{R_1+R_2+\beta}}$$