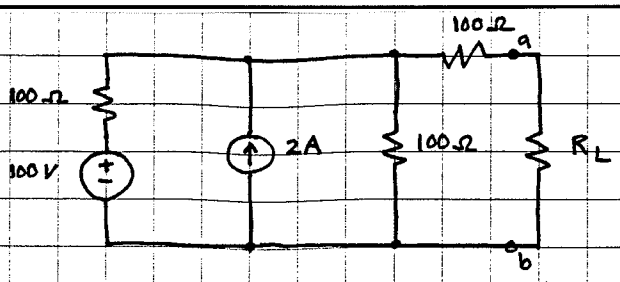
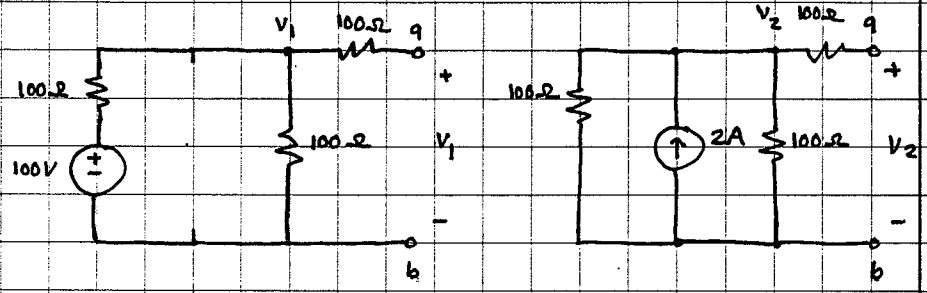
3.



a) Calculate the value of R_L that would absorb maximum power.

Soln: Transform into Thevenin equivalent of circuit left of R_L. Use superposition the find V_{Th}:



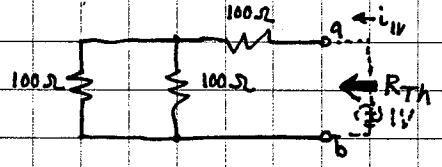
$V_{Th} = v_1 + v_2$ In both cases, we have no current thru 100Ω top right. So no V-drop across it.

v_1 from V-divider: $v_1 = 100V \cdot \frac{100\Omega}{100\Omega + 100\Omega} = 50V$

$v_2 = 2A \cdot 100\Omega \| 100\Omega = 2A \cdot 50\Omega = 100V$

$V_{Th} = v_1 + v_2 = 150V$

For R_{Th}, we turn (independent) sources to zero and see what resistance we have looking in from a-b terminals:



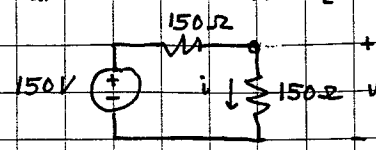
(or connect 1V source and use $1V/i_{1V}$)

$R_{Th} = 100\Omega + 100\Omega \| 100\Omega = 150\Omega$

Max power xfer when $\boxed{R_L = R_{Th} = 150\Omega}$

3.b) Calculate the value of max power, R_L chosen for max pwr.

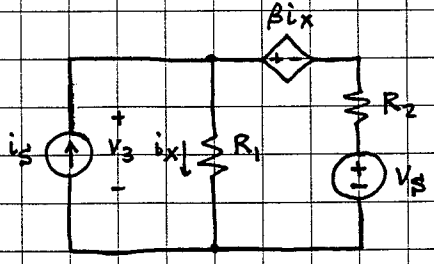
sol'n: Our circuit with $R_L = R_{Th}$ is:



pwr $p = iv = \frac{150V}{300\Omega} \cdot 75V$

$p = 37.5\ W$

4.

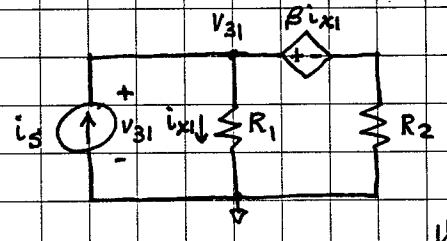


Use superposition. Find expression for v_3 not using i_x.

sol'n: Turn one independent source on at a time, with other independent sources = 0. Sum currents and v's from each of the models to get total currents and voltages for circuit.

Note: Current source set to 0 is open circuit.
Voltage " " " " " short circuit, (i.e. wire).

Circuit 1: $i_s \neq 0$, $v_s = 0$. Add a "1" subscript to i's and v's.



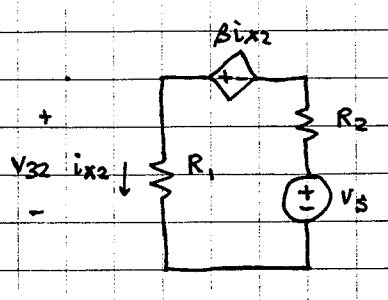
$$i_{x1} = \frac{V_{31}}{R_1}$$

Use Node V for V_{31}:

$$i_s = i_{x1} + \frac{V_{31} - \beta i_{x1}}{R_2} = \frac{V_{31}}{R_1} + \frac{V_{31} - \beta \frac{V_{31}}{R_1}}{R_2}$$

$$i_s = V_{31}\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\beta}{R_1 R_2}\right) \quad \text{or} \quad V_{31} = i_s \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\beta}{R_1 R_2}\right)$$

4. (cont) Circuit 2: $v_s \neq 0$, $i_s = 0$. Add a "2" subscript to i's & v's.



Use sum V's around loop = 0. (Mesh i)

$i_{x2} = \frac{v_{32}}{R_1}$

$v_s - i_{x2} R_2 + \beta i_{x2} - i_{x2} R_1 = 0V$ (v_{32} = $i_{x2} R_1$)

or $v_s - v_{32}\frac{R_2}{R_1} + \beta\frac{v_{32}}{R_1} - v_{32} = 0V$

$v_s = v_{32}\left(\frac{R_2}{R_1} + 1 - \frac{\beta}{R_1}\right)$ $\quad v_{32} = \dfrac{v_s}{\frac{R_2}{R_1} + 1 - \frac{\beta}{R_1}}$

Now $v_3 = v_{31} + v_{32}$ $= \dfrac{v_s}{R_2\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\beta}{R_1 R_2}\right)}$

eqn 1 or $v_3 = \left(i_s + \frac{v_s}{R_2}\right) \Big/\!\Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\beta}{R_1 R_2}\right)$

eqn 2 $\boxed{v_3 = \left(i_s + \frac{v_s}{R_2}\right)\left(R_1 \| R_2 \| -\frac{R_1 R_2}{\beta}\right)}$ Note: $R_a \| -R_b \equiv \frac{R_a R_b}{R_b - R_a}$

eqn 3 $v_3 = \left(i_s R_2 + v_s\right) \Big/\!\Big/ \left(\frac{R_2}{R_1} + 1 - \frac{\beta}{R_1}\right)$ $= \dfrac{R_a(-R_b)}{R_a + (-R_b)}$

Consistency Checks: 1) $\beta = 0$, $v_s = 0 \Rightarrow v_3 = i_s \cdot (R_1 \| R_2)$

eqn 2 gives $v_3 = \left(i_s + \frac{0}{R_2}\right)\left(R_1 \| R_2 \| \frac{-R_1 R_2}{0 \to \infty}\right) = i_s \cdot (R_1 \| R_2)$ ✓

2) $R_1 = \infty \Rightarrow v_3 = v_s + i_s R_2$

eqn 2 gives $v_3 = \left(i_s + \frac{v_s}{R_2}\right)\left(\infty \| R_2 \| \frac{\infty \cdot R_2}{\beta}\right) = \left(i_s + \frac{v_s}{R_2}\right) R_2$ ✓

3) $i_s = 0$, $\beta = 0 \Rightarrow v_3 = v_s \dfrac{R_1}{R_1 + R_2}$

eqn 2 gives $v_3 = \left(0 + \frac{v_s}{R_2}\right)\left(R_1 \| R_2 \| \frac{-R_1 R_2}{0 \to \infty}\right) = \frac{v_s}{R_2} \frac{R_1 R_2}{R_1 + R_2}$ ✓

4) $R_1 = 1\Omega$, $R_2 = 2\Omega$, $v_s = 3V$, $i_x = 4A$, $v_3 = 4V$, $\beta = \frac{1}{8}$,
$\beta i_x = \frac{1}{2}V$, $v_{R_2} = v_3 - (v_s + \beta i_x) = 4 - (3 + \frac{1}{2}) = \frac{1}{2}V \Rightarrow i_{R_2} = \frac{1}{4}A$.
$i_s = i_x + i_{R_2} = 4\frac{1}{4}A = 17/4A$ $R_1 \| R_2 \| -R_1 R_2/\beta = 1 \| 2 \| 16 = \frac{2}{3} \| 16 = \frac{1}{3} \cdot 2 \| 48 = +32/46$
eqn 2 gives $v_3 = \left(\frac{17}{4} + \frac{3}{2}\right) \cdot \frac{+32}{46} = \frac{23}{4} \cdot \frac{32}{46} = \frac{32}{8} = 4V$ ✓