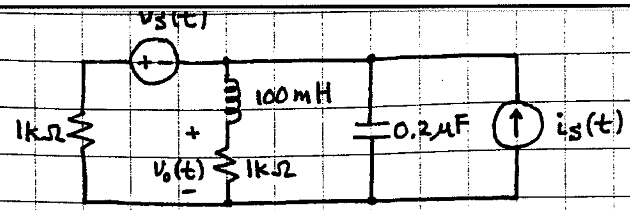
3.



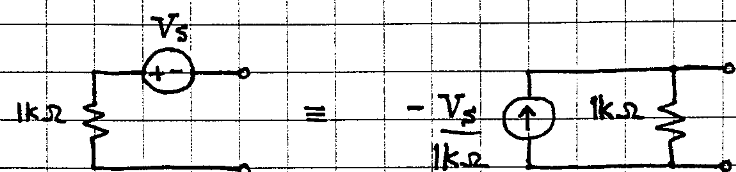
$v_s(t) = 100 \cos(10^4 t)$ V
$i_s(t) = \sin(10^4 t)$ A

a) Find a numerical, time-domain expression for $v_o(t)$.

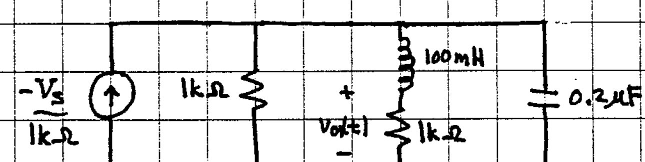
sol'n: I'll use superposition. Other approaches such as node-V or mesh current would work, too.

circuit 1: $V_s = 100\angle 0°$ V set $I_s = 0$ A

Convert V_s and 1kΩ on left to Norton equivalent:

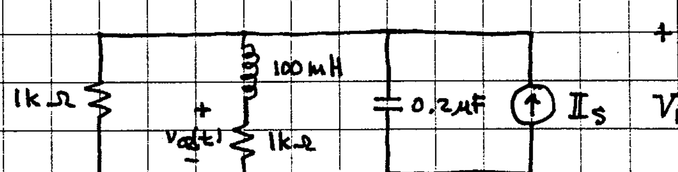


Our circuit 1 is now:



circuit 2: $V_s = 0$ V and $I_s = 1\angle -90°$ A (phasor for i_s)

Keeping the Norton equivalent, our circuit 2 is:



Clearly, we can sum the current sources from circuits 1 & 2 to find V_1 drop from top rail to bottom rail.

3.a(cont) $\quad \mathbb{V}_1 = \left(-\dfrac{\mathbb{V}_s}{1k\Omega} + \mathbb{I}_s \right) 1k\Omega \,\|\, (j\omega L + 1k\Omega) \,\|\, \dfrac{-j}{\omega C}$

$$j\omega L = j\,10^4\,100m = j\,1k\Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{10^4\,0.2\mu} = \frac{-j}{2\,m} = \frac{-j \cdot 1k\Omega}{2}$$

$$-\frac{\mathbb{V}_s}{1k\Omega} + \mathbb{I}_s = -\frac{100\angle 0^\circ\,V}{1k\Omega} + 1\angle -90^\circ A$$

$$= -0.1\angle 0^\circ A + 1\angle -90^\circ A$$

$$= -0.1 - j$$

$$= \sqrt{0.1^2 + 1^2}\ \angle \tan^{-1}\frac{-1}{-0.1}$$

$$= 1.005\ \angle -95.7^\circ$$

$$1k\Omega \,\|\, (j1k\Omega + 1k\Omega) \,\|\, -\frac{j}{2}1k\Omega = 1k\Omega \frac{1}{\frac{1}{1} + \frac{1}{1+j} - \frac{2}{j}}$$

$$= \frac{1k}{1 + \frac{1-j}{2} + 2j} = \frac{1k}{\frac{3}{2} + j\frac{3}{2}} = \frac{2}{3}\,\frac{1k}{1+j}$$

$$= \frac{2}{3}k\,\frac{1-j}{2} = 1k \cdot \frac{(1-j)}{3}$$

$$\mathbb{V}_1 = 1.005k\angle -95.7^\circ \cdot \frac{1-j}{3}\ V$$

$\mathbb{V}_o$ from v-divider:

$$\mathbb{V}_o = \mathbb{V}_1 \cdot \frac{1k\Omega}{1k\Omega + j\omega L} = \mathbb{V}_1 \cdot \frac{1k\Omega}{1k\Omega + j1k\Omega} = \mathbb{V}_1 \frac{1}{1+j} = \mathbb{V}_1 \frac{(1-j)}{2}$$

$$\mathbb{V}_o = 1.005k\angle -95.7^\circ \left(\frac{1-j}{3}\,\frac{1-j}{2} = \frac{-j2}{6} \right) = \frac{1.005k\angle -185.7^\circ}{3}$$

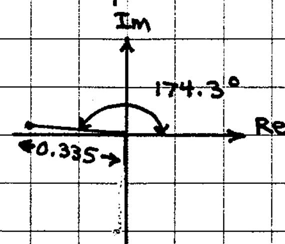
3a (cont) $\quad \mathbf{V}_o = 0.335\text{k}\angle -185.7^\circ \text{ V}$

$$v_o(t) = 0.335\text{k}\cos(10^4 t - 185.7^\circ) = 0.335\text{k}\cos(10^4 t + 174.3^\circ) \text{ V}$$

or $\quad v_o(t) = -0.335\text{k}\cos(10^4 t - 5.7^\circ) \text{ V}$

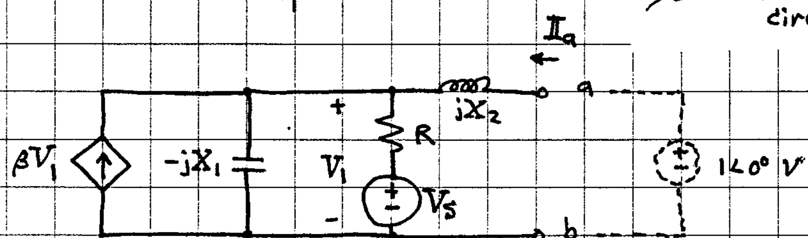
b) Show $\mathbf{V}_o$ on a phasor diagram.

soln:



$\mathbf{I}_a$ flows into circuit

4.



Construct a frequency domain Thevenin equivalent.

soln: $\mathbf{V}_{Th}$ = open circuit output $\mathbf{V}$ (phasor) $= \mathbf{V}_1$ (since no drop across jX_2)

Use Node-V: $\quad \dfrac{\mathbf{V}_1 - \mathbf{V}_s}{R} + \dfrac{\mathbf{V}_1}{-jX_1} - \beta \mathbf{V}_1 = 0 \text{ A}$

$$\mathbf{V}_1 \left(\frac{1}{R} - \frac{1}{jX_1} - \beta \right) = \frac{\mathbf{V}_s}{R}$$

$$\mathbf{V}_{Th} = \mathbf{V}_1 = \mathbf{V}_s \frac{\left(R \parallel -jX_1 \parallel -\frac{1}{\beta}\right)}{R} \quad \text{or} \quad \mathbf{V}_s \left(1 \parallel \frac{jX_1}{R} \parallel \frac{1}{R\beta} \right)$$

$$z_{Th} = \frac{1\angle 0^\circ \text{ V}}{\mathbf{I}_a} \quad \text{with } \mathbf{V}_s \text{ set to } 0 \text{ V}$$

Use Node-V: $\quad \dfrac{\mathbf{V}_1 - 1\angle 0^\circ \text{V}}{jX_2} + \dfrac{\mathbf{V}_1}{R} + \dfrac{\mathbf{V}_1}{-jX_1} - \beta \mathbf{V}_1 = 0 \text{ A}$

$$\mathbf{V}_1 \left(\frac{1}{jX_2} + \frac{1}{R} - \frac{1}{jX_1} - \beta \right) = \frac{1\angle 0^\circ \text{V}}{jX_2}, \quad -\mathbf{I}_a = \frac{\mathbf{V}_1 - 1\angle 0^\circ \text{V}}{jX_2}$$

$$z_{Th} = \frac{-jX_2 \, 1\angle 0^\circ \text{V}}{\mathbf{V}_1 - 1\angle 0^\circ \text{V}} = -jX_2 \Big/ \left(\left(1 \parallel \frac{R}{jX_2} \parallel -\frac{X_1}{X_2} \parallel \frac{-1}{jX_2\beta} \right) - 1 \right)$$