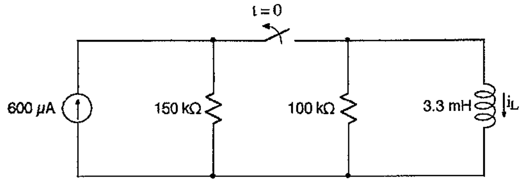
Problem Set #6

ECE 1270
Sp 06

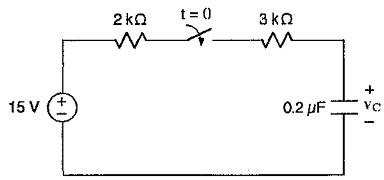
~~Problem Session #4~~
HOMEWORK #5

1.



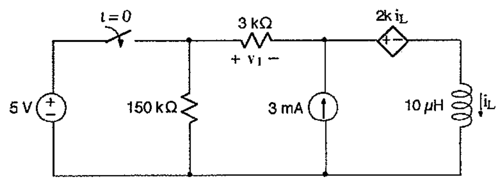
After being closed for a long time, the switch opens at t = 0. Find $i_L(t)$ for t > 0.

2.



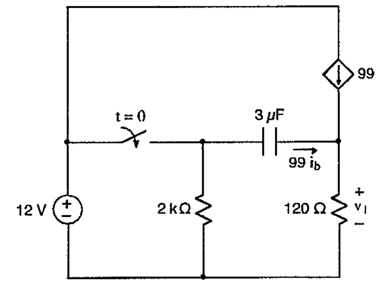
After being open for a long time, the switch closes at t = 0. $v_C(t = 0^-) = 0V$.
Find $v_C(t)$ for t > 0.

3.



After being open for a long time, the switch closes at t = 0. Find $v_1(t)$ for t > 0.

4.



After being open for a long time, the switch closes at t = 0. Find $v_1(t)$ for t > 0.

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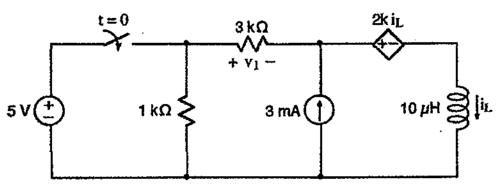
HOMEWORK #5 Solution Prob 3

Ex:



After being open for a long time, the switch closes at t = 0. Find $v_1(t)$ for t > 0.

soln: To find $v_1(t>0)$, we use the general form of solution, (which applies to any current or voltage):

$$v_1(t>0) = v_1(t\to\infty) + [v_1(0^+) - v_1(t\to\infty)]\, e^{-t/\frac{L}{R_{Th}}}$$

We have an inductor whose behavior at time $t=0^+$ will affect the value of $v_1(0^+)$.

We find the value of i_L at time $t=0^-$ and employ the concept that i_L, being an energy variable, cannot change instantly. Thus, $i_L(0^+) = i_L(0^-)$.

At $t=0^+$, currents and voltages have stabilized, and time derivatives $= 0$. Thus, $v_L = L\,di/dt = 0V$ and the L acts like a wire: it has no v drop but it can carry current.

At $t=0^-$, the switch is open, removing the 5V source from the circuit.

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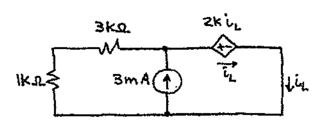
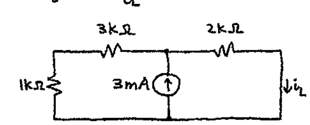
HOMEWORK #5 Solution Prob 3 (cont.)

$t=0^-$:



We observe that the dependent source is equivalent to a resistor.

$$R_{eq} = \frac{v}{i} = \frac{2k\, i_L}{i_L} = 2k\Omega$$



This is a current divider.

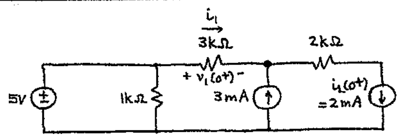
$$i_L(0^-) = 3\text{ mA}\,\frac{1k\Omega + 3k\Omega}{1k\Omega + 3k\Omega + 2k\Omega} = 2mA$$

Note that we do not find $v_1(0^-)$ since it may change instantly when the switch closes.

$t=0^+$: We model the L as a current source with $i_L(0^+) = i_L(0^-)$.

The switch is closed for $t>0$.

As before the dependent source acts like a $2k\Omega$ resistor.



A current summation at the node shown as a large dot gives the current thru the 3kΩ resistor:

$$-i_1 - 3mA + 2mA = 0\ A$$

or

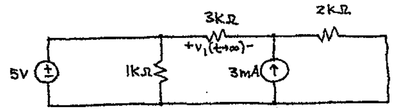
$$i_1 = -1\ mA$$

By Ohm's law, we have

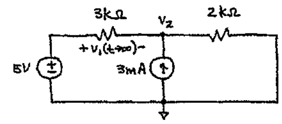
$$v_1(0^+) = i_1 \cdot 3k\Omega = -1mA \cdot 3k\Omega = -3V$$

Now we find $v_1(t\to\infty)$.

$t\to\infty$: The L again acts like a wire, and the switch is closed.



We may ignore the 1kΩ resistor since it acts like a separate circuit across the 5V source. (The other circuit across the 5V source consists of 3kΩ, 2kΩ, and i src.



Using the node-voltage method, we have

$$\frac{v_2 - 5V}{3k\Omega} - 3mA + \frac{v_2}{2k\Omega} = 0\ A$$

Multiplying both sides by 6kΩ yields

$$v_2\,(2+3) = 2\cdot 5V + 6k\Omega \cdot 3mA$$

or

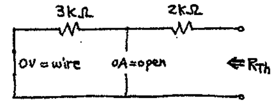
$$5v_2 = 28V$$

or

$$v_2 = \frac{28V}{5}$$

Thus, $v_1(t\to\infty) = 5V - v_2 = 5V - \frac{28V}{5} = -\frac{3}{5}V$

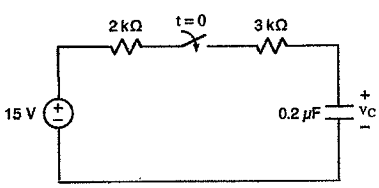
R_{Th}: We can use the circuit at the top of the page with the independent sources set to zero and the L (i.e., wire) removed.



We have
$R_{Th} = 2k\Omega + 3k\Omega$

$$\therefore v_1(t>0) = -\frac{3}{5}V + \left[-3V - -\frac{3V}{5}\right] e^{-\frac{t}{10H/5k}} = -0.6 - 2.4e^{-t/50ms}\ V$$

Ex:



After being open for a long time, the switch closes at t = 0. $v_C(t = 0^-) = 0V$.
Find $v_C(t)$ for t > 0.

sol'n: Use the general form of solution for RC problems.

$$v_c(t>0) = v_c(t\to\infty) + [v_c(0^+) - v_c(t\to\infty)]\, e^{-t/R_{Th}C}$$

We now proceed to find the following values:

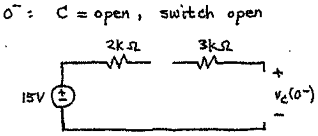
$v_c(0^+)$, $v_c(t\to\infty)$, and R_{Th}

To find $v_c(0^+)$, we consider $t=0^-$ and find $v_c(0^-)$. Since v_c is an energy variable that cannot change instantly, we have $v_c(0^+) = v_c(0^-)$.

At $t=0^-$, currents and voltages have stabilized, and all time derivatives of currents and voltages are zero.

Thus, $i_c = C\dfrac{dv_c}{dt} = C\cdot 0 = 0$. C looks like open.

$t=0^-$: C = open, switch open



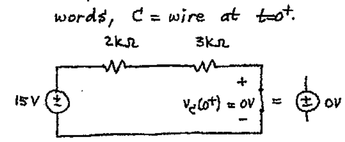
From the circuit diagram, we cannot determine $v_c(0^-)$. The C could be charged to some voltage, and it would remain at that voltage forever.

Fortunately, the problem states that $v_c(0^-) = 0V$.

$t=0^+$: v_c cannot change instantly, so

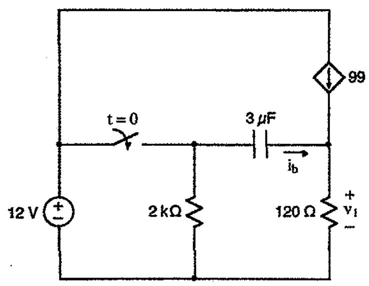
$$v_c(0^+) = v_c(0^-) = 0V$$

If needed a circuit model at $t=0^+$, we would model the C as a v src with value 0V. In other words, C = wire at $t=0^+$.



To find $v_c(t\to\infty)$, we again use the idea that currents and voltages are stable and C = open.

Ex:

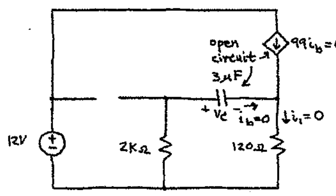


After being open for a long time, the switch closes at t = 0. Find $v_1(t)$ for t > 0.

soln: Use general form of solution for RC problems:

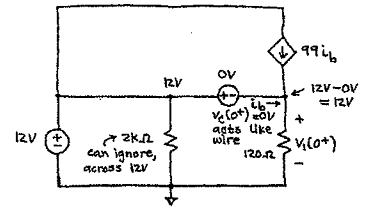
$$v_1(t>0) = v_1(t\to\infty) + \left[v_1(0^+) - v_1(t\to\infty)\right] e^{-t/R_{Th}C}$$

$t=0^-$: C acts like open circuit $\Rightarrow i_b = 0$, $99 i_b = 0$
switch is open



Since no power is connected to C, $v_C(0^-) = 0V$

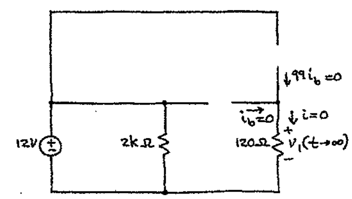
$t=0^+$: C acts like v src, $v_C(0^+) = v_C(0^-)$.
switch is closed



If we consider a v-loop around the outside of the bottom half of the circuit, we find that we have 12V across the 120Ω resistor:

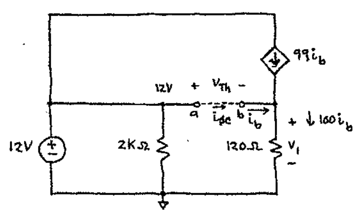
$$v_1(0^+) = 12V$$

$t\to\infty$: C acts like open circuit $\Rightarrow i_b = 0$, $99i_b = 0$
switch is closed



No power is connected to 120Ω.
Thus, $v_1(t\to\infty) = 0V$

R_{Th}:



$R_{Th} = \dfrac{V_{Th}}{i_{sc}}$ $\qquad V_{Th} = V_{ab}$ with C removed (a,b open circuit)

With open circuit a,b we have $i_b = 0$ and $99 i_b = 0$. Thus, $v_1 = 0V$.

$$V_{Th} = 12V - v_1 = 12V - 0V = 12V$$

Now connect wire from a to b and measure current, i_{sc}.

We have $v_1 = 12V$ since it is now connected across 12V source by wires.

The current thru the 120Ω resistor is $100 i_b$:

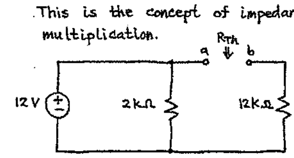
$$\frac{v_1}{120\Omega} = \frac{12V}{120\Omega} = 100\,mA = 100\, i_b$$

Thus, $i_{sc} = i_b = \dfrac{100mA}{100} = 1\,mA$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{12V}{1\,mA} = 12k\Omega$$

Note: We get the same result if we remove the dependent source and multiply the 120Ω resistor by 100 to account for the $100 i_b$ flowing thru it.

This is the concept of impedance multiplication.



We find R_{Th} by turning off the 12V source (which becomes a wire) and determining R seen looking into a,b.

We have $R_{Th} = 12k\Omega$, as before.

Plugging values into the general solution yields our final answer:

$$v_1(t) = 0V + \left[12V - 0V\right] e^{-t/12k\Omega\cdot 3\mu F}, \qquad t>0$$

or

$$v_1(t>0) = 12V\, e^{-t/36ms}$$