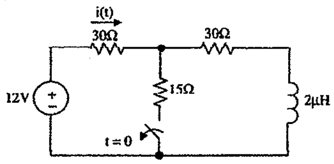
Problem Set #7

ECE 1000 Exam #3 Sol'n Sp '04

1. (30 points)

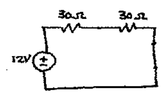


After being closed for a long time, the switch is <u>opened</u> at t = 0.

a. Calculate the energy stored on the inductor at $t \to \infty$.

b. Write a numerical expression for $i(t)$, $t > 0$.

soln: a) $t \to \infty$: model L as wire, switch open



$i_L(t\to\infty) = \frac{12V}{30+30\Omega} = \frac{12V}{60\Omega}$

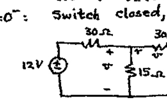
$\quad " \quad = \frac{1}{5}A$

Energy $w_L(t\to\infty) = \frac{1}{2} L i_L^2(t\to\infty) = \frac{1}{\cancel{2}} \cancel{2}\mu H \cdot \left(\frac{1}{5}A\right)^2$

$$\boxed{w_L(t\to\infty) = \frac{1}{25}\mu W \text{ or } 40nW}$$

Note: $i(t\to\infty) = i_L(t\to\infty) = \frac{1}{5}A = 200mA$

b) $t = 0^-$: Switch closed, L = wire, find $i_L(0^-)$



$i_L(0^-) = \frac{v}{30\Omega}$

$v = \frac{15\|30\Omega}{15\|30 + 30\Omega} \cdot 12V$

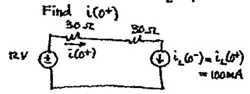
$t = 0^+$: switch open, L = current src $= i_L(0^-)$

$15\|30\Omega = 15\Omega \cdot 1\|2 = 15\Omega \cdot \frac{2}{3} = 10\Omega$

$v = \frac{10}{10+30} \cdot 12V = 3V$

Find $i(0^+)$

$i_L(0^-) = \frac{3V}{30\Omega} = 100mA$

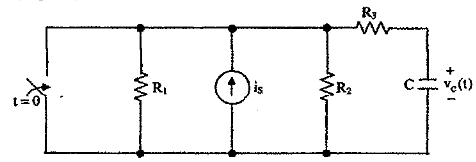


$i(0^+) = i_L(0^+) = i_L(0^-) = 100mA$

$0 < t < \infty$: R_{Thev} of circuit $= 30\Omega + 30\Omega = 60\Omega$ $\therefore \tau = \frac{L}{R_{Th}} = \frac{2\mu H}{60\Omega} = \frac{1}{30}\mu s$

$$\boxed{i(t>0) = i(t\to\infty) + [i(0^+) - i(t\to\infty)]e^{-t/\tau} = 200 - 100e^{-t/\frac{1}{30}\mu s}\ mA}$$

2. (25 points)

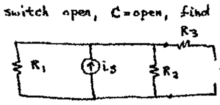


After being open for a long time, the switch is <u>closed</u> at t = 0.

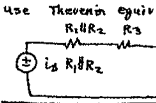
a. Write an expression for $v_c(t = 0^+)$.

b. Write an expression for $v_c(t)$, $t > 0$.

sol'n: a) $t = 0^-$: switch open, C = open, find $v_C(0^-)$ since $v_C(0^+) = v_C(0^-)$



use Thevenin equiv of i_s, R_1, and R_2

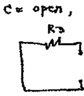


$v_C(0^-) = i_s \cdot R_1\|R_2 = v_C(0^+)$

$$\boxed{v_C(0^+) = i_s \cdot R_1\|R_2 \text{ or } i_s \frac{R_1R_2}{R_1+R_2}}$$

b) $v_c(0^+) = i_s \cdot R_1\|R_2$ so move on, to $t\to\infty$ and R_{Thev}

$t\to\infty$: switch closed (so we can ignore R_1, i_s, and R_2), C = open, find $v_C(t\to\infty)$



$v_C(t\to\infty) = 0V$

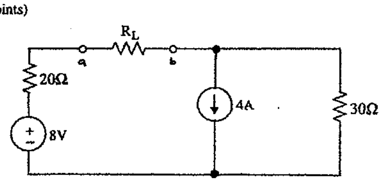
$0 < t < \infty$: from circuit for $t\to\infty$ we see that R_{Thev} of circuit where C is connected is just R_3

$\tau = R_{Th}C = R_3C$

$v_C(t\geq 0) = v_C(t\to\infty) + [v_C(0^+) - v_C(t\to\infty)]e^{-t/\tau}$

$$\boxed{v_C(t\geq 0) = i_s \cdot R_1\|R_2\, e^{-t/R_3C}}$$

3. (20 points)

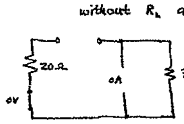


a. Calculate the value of R_L that would absorb maximum power.

b. Calculate that value of maximum power R_L could absorb.

soln: a) $R_L = R_{Thev}$ of circuit where R_L connected gives max pwr xfer

R_{Thev} = R looking into circuit at terminals a,b without R_L and src's = 0 (i.e. 8V=0V=wire, 4A=0A=open)

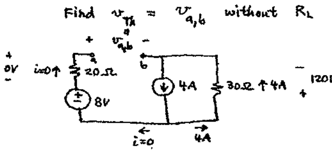


$R_{Th} = 20 + 30\Omega = 50\Omega$

$\therefore \boxed{R_L = 50\Omega}$

b) max $p = \frac{v_{Th}^2}{4R_{Th}}$

Find $v_{Th} = v_{a,b}$ without R_L

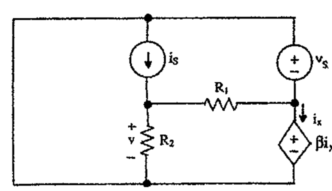


From outer v-loop, we have $v_{Th} = -8V - 120V = -128V$

max $p = \frac{(-128V)^2}{4 \cdot 50\Omega} = 32 \cdot \frac{128}{50} W = \frac{32 \cdot 128 \cdot 20}{1k} W = \frac{81.92k W}{1k}$

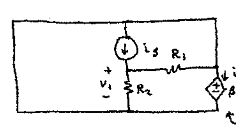
$$\boxed{\text{Max } p = 81.92\ W}$$

4. (25 points)



Using superposition, derive an expression for v that contains no circuit quantities other than i_s, v_S, R_1, R_2, and β, where $\beta > 0$.

sol'n: case I: i_s on, v_s off



$R_{eq} = \frac{\beta i_x}{i_x} = \beta\ \Omega$

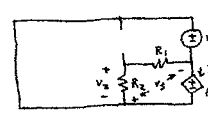
wire around outside shorts $R_{eq} \Rightarrow \beta i_x = 0V \Rightarrow i_x = 0$

$\therefore$ ignore dependent src

i-divider with R_1 and R_2 in parallel

$v_1 = i_s \cdot R_1\|R_2$

case II: i_s off, v_s on



$R_{eq} = \frac{\beta i_x}{i_x} = \beta\Omega$ as before

v-divider with v_s across R_1 and R_2 in series

$v_2 = -v_s \frac{R_2}{R_1+R_2}$

$$\boxed{v = v_1 + v_2 = i_s \cdot R_1\|R_2 - v_s \frac{R_2}{R_1+R_2}}$$