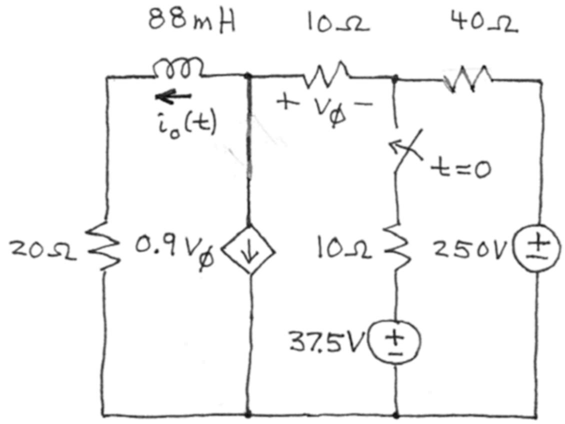
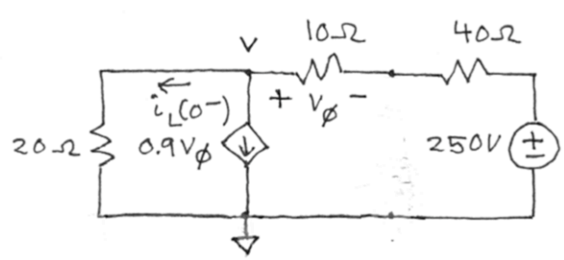
Ex:



After being open for a long time, the switch closes at $t=0$. Find $i_o(t \geq 0)$.

sol'n: $t=0^-$ ⇒ switch open, L acts like wire, find energy variable $i_L(0^-) = i_o(0^-)$. The energy variable does not change instantly, so $i_L(0^+) = i_L(0^-)$. All other i's and v's may change when the switch closes.



Using the node-voltage method, we find v:

$$\frac{v}{20\,\Omega} + 0.9(v - 250\,\text{V})\frac{10\,\Omega}{10\,\Omega + 40\,\Omega} + \frac{v - 250\,\text{V}}{10\,\Omega + 40\,\Omega} = 0\,\text{A}$$

$$v\left(\frac{1}{20\Omega} + \frac{0.9(10\Omega)+1}{10\Omega+40\Omega}\right) = \frac{[0.9(10\Omega)+1]}{10\Omega+40\Omega}250V$$

or

$$v\left(\frac{1}{20\Omega} + \frac{10}{50\Omega}\right) = \frac{10}{50\Omega}\ 250V$$

or

$$v\ (1+4) = 4\ (250V) \qquad \text{(Mult by } 20\Omega \text{ on both sides)}$$

or

$$v = 200V$$

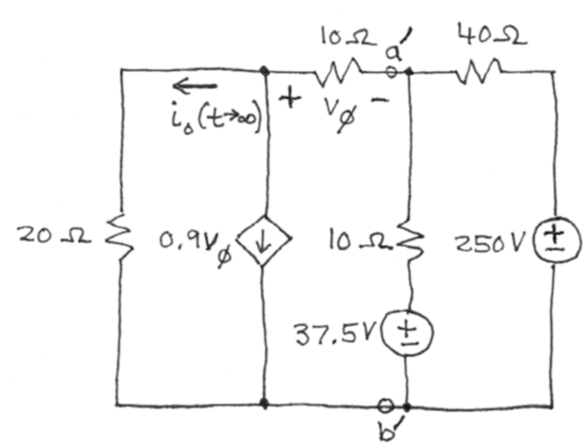
Using v, we find $i_L(0^-)$:

$$i_L(0^-) = \frac{200V}{20\Omega} = 10A$$

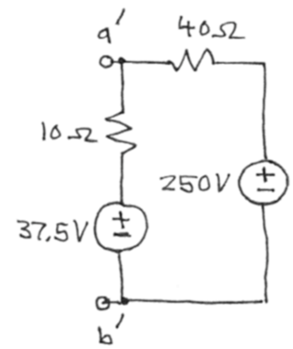
$t=0^+ \Rightarrow$ L acts like current source $i_L(0^+)=i_L(0^-)$ or $i_L = 10A$, switch is closed.

The variable we are interested in is $i_o(0^+) = i_L(0^+) = 10A$.

$t\to\infty \Rightarrow$ switch is closed, L acts like wire

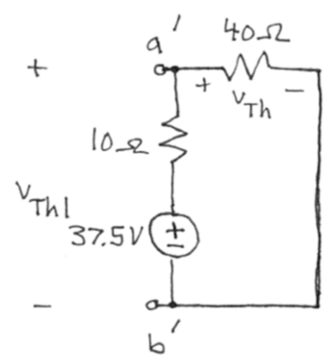


We achieve some simplification by replacing the circuit to the right of a′, b′ with its Thevenin equivalent.



The Thevenin voltage is the open circuit voltage across a′, b′. We use superposition to find v_{Th}:

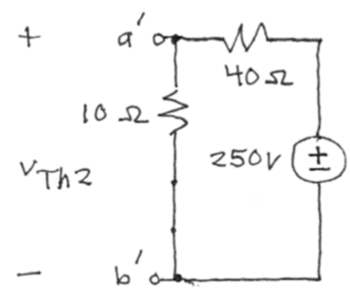
case I:
37.5V on
250V off



v_{Th1} = voltage across 40Ω

$$v_{Th1} = 37.5V \cdot \frac{40\Omega}{10\Omega + 40\Omega} = 30V$$

case II:
250V on
37.5V off

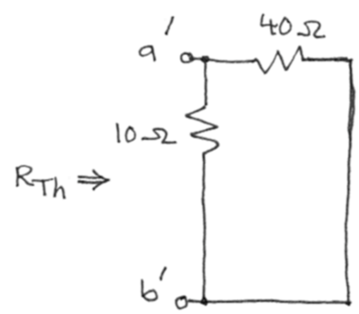


$$V_{Th2} = 250V \cdot \frac{10\Omega}{10\Omega + 40\Omega} = 50V$$

Summing the results gives V_{Th}:

$$V_{Th} = V_{Th1} + V_{Th2} = 30V + 50V = 80V$$

To find R_{Th}, we turn off the independent sources and look into the circuit from a', b':

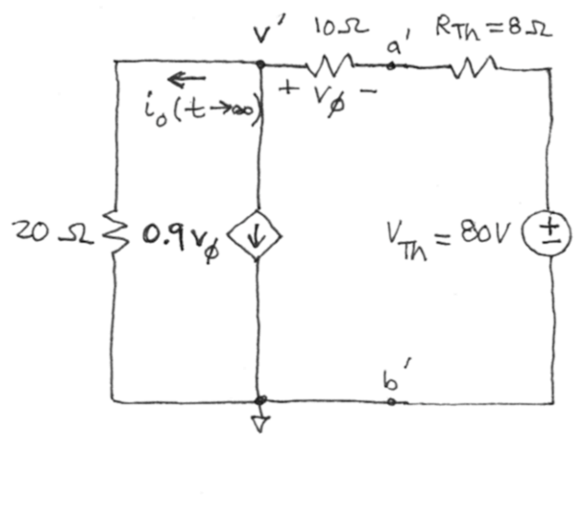


$$R_{Th} = 10\Omega \| 40\Omega = 10\Omega \cdot 1\|4 = 10\Omega \cdot \frac{1(4)}{1+4}$$

or

$$R_{Th} = 8\Omega$$

Our circuit now appears as follows:



Using the node-voltage method, we find v'.

$$\frac{v'}{20\Omega} + 0.9(v' - 80V)\frac{10\Omega}{10\Omega + 8\Omega} + \frac{(v' - 80V)}{10\Omega + 8\Omega} = 0A$$

or

$$v'\left(\frac{1}{20\Omega} + \frac{0.9(10\Omega)}{18\Omega} + \frac{1}{18\Omega}\right) = 80V\left(\frac{0.9(10\Omega)}{18\Omega} + \frac{1}{18\Omega}\right)$$

or

$$v'(9 + 90 + 10) = 80V\,(90 + 10) \quad \text{(mult by } 180\Omega)$$

or

$$v' = \frac{8000\,V}{109}$$

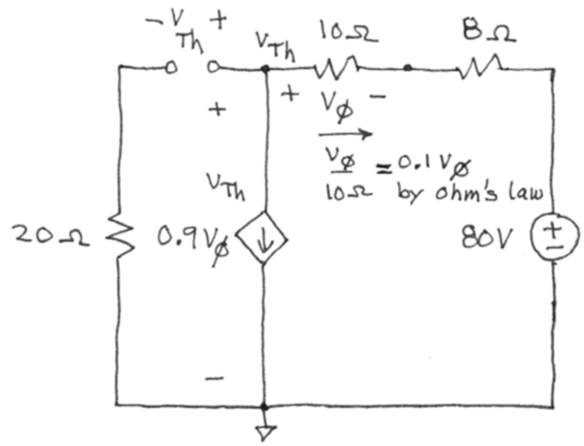
Using v', we find $i_o(t \to \infty)$:

$$i_o(t \to \infty) = \frac{v'}{20\Omega} = \frac{8000\,V}{109 \cdot 20\Omega} = \frac{400}{109}\,A$$

time const: $\tau = L / R_{Th}$ where R_{Th} is the Thevenin R seen from the terminals where the L is connected.

We find $R_{Th} = v_{Th} / i_{sc}$ where v_{Th} is the open-circuit voltage where the L is connected, and i_{sc} is the short-circuit current when the L is replaced by a wire. Here, $i_{sc} = i_o(t \to \infty) = \frac{400}{109}\,A$.

For v_{Th}, we have the following circuit:



Since no current flows in the 20Ω, v_{Th} is the same as the node-voltage labelled v_{Th} above. We can use the node-voltage method to find v_{Th}:

$$0.9(v_{Th} - 80V)\frac{10\Omega}{10\Omega+8\Omega} + \frac{(v_{Th} - 80V)}{10\Omega+18\Omega} = 0A$$

or

$$v_{Th}\left(\frac{0.9(10\Omega)}{18\Omega} + \frac{1}{18\Omega}\right) = 80V\left(\frac{0.9(10\Omega)}{18\Omega} + \frac{1}{18\Omega}\right)$$

or

$$v_{Th}(9+1) = 80V(9+1)$$

or

$$v_{Th} = 80V$$

Note: This result implies that $v_\phi = 0V$. This makes sense since, as indicated in the circuit diagram, the current in the loop on the right side of the circuit is $0.9 v_\phi = \frac{-v_\phi}{10\Omega} \Rightarrow v_\phi = 0$.

We have $\tau = \dfrac{L}{R_{Th}}$ where $R_{Th} = \dfrac{v_{Th}}{i_{sc}}$

$$R_{Th} = \frac{80V}{\frac{400}{109}\,A} = \frac{109}{5}\,\Omega$$

$$\tau = \frac{88\,mH}{\frac{109}{5}\,\Omega} = \frac{440}{109}\,ms \approx 4\,ms$$

To complete the problem, we use the general form of solution:

$$i_o(t \geq 0) = i_L(t\to\infty) + \left[i_o(0^+) - i(t\to\infty)\right]e^{-t/\tau}$$

or

$$i_o(t \geq 0) \doteq \frac{400}{109}\,A + \left(10 - \frac{400}{109}\,A\right)e^{-t/4\,ms}$$

or

$$i_o(t \geq 0) \approx 3.67A + 6.33Ae^{-t/4.04\,ms}$$