Design of Nonbinary LDPC Codes for Multiple-Antenna Transmission

Rong-Hui Peng and Rong-Rong Chen

Department of Electrical and Computer Engineering
University of Utah

This work is supported in part by NSF under grant ECS-0547433.
Outline

• Motivation
• Two nonbinary LDPC coded system
  – Non-iterative system
  – Iterative system
• Code design for iterative system
• Simulation results
• Conclusion
Motivation

- MIMO system has been widely used to increase system capacity
- LDPC codes can be employed to approach the MIMO channel capacity
  - S. ten Brink, G. Kramer, and A. Ashikhmin study the binary LDPC code design for MIMO [1].
- Nonbinary LDPC code design has been studied for AWGN and shows better performance than binary codes [2].


Motivation

- Our contribution
  - The first work to apply irregular nonbinary LDPC to MIMO channel
  - Propose two nonbinary LDPC coded MIMO systems.
  - Extend EXIT chart based code design methods to nonbinary iterative systems
  - Provide comparison with optimal binary LDPC coded systems
Introduction of binary LDPC

• A subclass of linear block codes
• Specified by a parity check matrix \((n-k) \times n\)

\(n\): code length \(k\): length of information sequence

\[
\begin{align*}
\mathbf{H} &= \begin{bmatrix}
1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 1
\end{bmatrix} \\
\mathbf{H}^T \mathbf{x} &= 0
\end{align*}
\]

\(c_1: x_1 + x_2 + x_3 + x_5 = 0\)
\(c_2: x_2 + x_3 + x_4 + x_6 = 0\)
\(c_3: x_1 + x_3 + x_4 + x_7 = 0\)
Definition of nonbinary LDPC

- For nonbinary codes, the ones in parity check matrix are replaced by nonzero elements in GF($q$)

\[
H = \begin{bmatrix}
3 & 7 & 1 & 0 & 3 & 0 & 0 \\
0 & 2 & 5 & 6 & 0 & 3 & 0 \\
4 & 0 & 2 & 7 & 0 & 0 & 5
\end{bmatrix}
\]
• Channel model

\[ X = \sqrt{\frac{\rho}{M}} HS + V \]

Assume each entry of channel matrix is independent, follows Rayleigh fading, and is known by receiver
Iterative system: Soft messages are exchanged between detector and decoder iteratively.

Non-iterative system: the detection is performed only once.

Code design for iterative system

- Irregular codes can achieve much better performance than regular codes.
- Density evolution and EXIT chart are two widely used methods to design binary irregular codes.
- So far, design of nonbinary codes is limited to AWGN channel.
- We focus on the code design for iterative MIMO system based on EXIT chart method.
- The idea of EXIT chart is tracking the soft message changed during iterations using mutual information.
Binary LDPC code design

Three types of soft messages

– Channel message
– Variable to Check message
– Check to Variable message

• Assumption: All messages are Gaussian distributed
  \[ N(\frac{\sigma^2}{2}, \sigma^2) \]

• EXIT function describe the relation of input message and output message in terms of mutual information. \( I_{out}(I_{in}) \)
Variable to Check Message

Repetition code

Variable node decoder (VND)

VND EXIT function:

\[ I_{E,VND}(I_A, d_v, SNR) = J(\sqrt{(d_v - 1)[J^{-1}(I_A)]^2 + \sigma_{ch}^2}) \]

\( J(\sigma) \) is the mutual information of AWGN channel with variance \( \sigma^2 \)

\( d_v \) is the degree of the variable node
Check to Variable Message

Single parity check code

\[ CND \text{ EXIT function:} \]

\[ I_{E,CND}(I_A, d_c) \approx 1 - J\left(\sqrt{(d_c - 1) \cdot J^{-1}(1 - I_A)}\right) \]

\( d_c \) is the degree of check node

Check node decoder (CND)
Code design using EXIT chart

- EXIT function for MIMO Detector can be evaluated by Monte Carlo simulation
- Construct combined EXIT function of detector and VND

\[ I_{E,VND/DET}(I_{A,VND/DET}, d_v, SNR) \]

- The EXIT function of irregular codes is a linear combination of the EXIT function of regular codes
Code design using EXIT chart

- To ensure successful decoding, VND/DET EXIT curve should lie above CND EXIT curve
- To approach capacity, VND/DET EXIT curve should match CND EXIT curve
- Code design can be done by curve matching

Challenges of nonbinary code design

- Soft message in binary system is LLR.

$$\ln \frac{p(b = 0)}{p(b = 1)}$$

- Soft message in nonbinary system is a vector-LLRV denote the log-likelihood ratio of being one element in GF(q).

$$z = \{z_0, z_1, \cdots, z_{q-1}\}$$

where $$z_i = \ln \frac{p(\beta = 0)}{p(\beta = i)}$$

$$i \in \{0,1,\cdots,q-1\}$$
Challenges of nonbinary code design

• It is more complex to evaluate the mutual information because of the multi-dimensional integration of the soft message

• Only sum of check to variable messages can be modeled as Gaussian vector with mean \( \mathbf{m} \) and variance \( \Sigma \)

\[
\mathbf{m} = \begin{bmatrix}
\sigma^2/2 \\
\sigma^2/2 \\
\vdots \\
\sigma^2/2
\end{bmatrix} \quad \Sigma = \begin{bmatrix}
\sigma^2 & \sigma^2/2 \\
\sigma^2/2 & \sigma^2 \\
\vdots & \vdots \\
\sigma^2/2 & \sigma^2
\end{bmatrix}
\]

• Other messages may be evaluated by Monte Carlo simulation
Generating of the EXIT curves using open-loop system

Input: $d_v, d_c, \sigma^2$, received vector $y$, coset vector $v$

$w_1$ and $w_2$ are Gaussian vector with $\sigma_1^2 = \sigma_c^2, \sigma_2^2 = \sigma_v^2 \frac{d_v}{d_v - 1}$. Model the sum of check to variable message
Generating of the EXIT curves using open-loop system

- For each \((d_v, d_c)\) We compute:
  
  \[ I_{E,VND/DET1}(I_{A,VND/DET1}, d_v, d_c) \]
  
  \[ I_{E,VND/DET2}(I_{A,VND/DET2}, d_v, d_c) \]
  
  \[ I_{E,CND}(I_A, d_v, d_c) \]

- \( I_{E,VND/DET2}(I_{A,VND/DET2}, d_v, d_c) \) is more accurate, since no Gaussian assumption is made.
Generating of the EXIT curves using open-loop system

• Comparison of two VND/DET EXIT curves
Simulation results

GF(16) lowest complexity, better performance, than GF(2)
GF(256) best performance, close complexity with GF(2)
Conclusion

- Application of nonbinary LDPC codes for iterative and noniterative MIMO system are studied.
- Code design for nonbinary LDPC coded iterative system is proposed.
- Nonbinary LDPC codes achieve better performance and lower complexity than the optimal binary LDPC codes and therefore are good candidate of MIMO system.
- Future work
  - Low complexity decoding algorithm
  - Code construction with linear encoding
Thanks !