Recent Advances in FDTD Modeling of Electromagnetic Wave Propagation in the Ionosphere

Bach T. Nguyen, Alireza Samimi, and Jamesina J. Simpson

Department of Electrical and Computer Engineering
University of Utah, Salt Lake City, UT 84112, USA
bach.nguyen@utah.edu, ar.samimi@utah.edu, jamesina.simpson@utah.edu

Abstract **─** Finite-Difference Time-Domain (FDTD) modeling of electromagnetic wave propagation in the Earth-ionosphere waveguide has gained significant interest over the past two decades. Initially, FDTD modeling capabilities were largely limited to two-dimensional models assuming a plasma ionosphere (but incapable of accounting for Faraday rotation), or to three-dimensional global models assuming a simple, isotropic conductivity profile ionosphere. Two algorithm developments have recently advanced the state-of-the-art in electromagnetic wave calculation capabilities in the ionosphere:

(1) A new, three-dimensional efficient FDTD magnetized plasma model.
(2) A Stochastic FDTD (S-FDTD) model of magnetized ionospheric plasma.

The first capability permits longer-distance, higher frequency and higher altitude propagation studies by greatly reducing the memory requirements and simulation time relative to previous plasma models. The second capability introduces for the first time a way of solving for not only mean electromagnetic field values, but also their variance. This paper provides an overview of these two recent advances.

Index Terms **─** Earth, electromagnetic wave propagation, Finite-Difference Time-Domain (FDTD), global propagation, ionosphere, magnetized plasma, plasma, stochastic processes.

I. INTRODUCTION

Many communications, radar, and geophysical studies and applications rely on accurate knowledge of both the state of the ionosphere and the characteristics of Electromagnetic (EM) signal propagation through or reflected by the ionosphere. Satellite communications, the Global Positioning System (GPS), over-the-horizon radar, target direction finding, and ionospheric remote sensing are some example applications. The success of these applications would be greatly improved with the availability of accurate modeling capabilities. Three major challenges, however, must be overcome in order to perform realistic calculations of EM propagation through the ionosphere:

(1) For most applications, the EM wave frequency is high enough such that complex magnetized plasma physics must be accommodated.
(2) The ionosphere exhibits high variability and uncertainty in both time and space.
(3) The ionosphere is comprised of both large and small-scale structures that often need to be accommodated.

Several approximate methods involving ray tracing have been proposed to calculate trans-ionospheric EM wave propagation (e.g., [1-4]); however, these methods are incapable of taking into account the full ionospheric variability and/or terrain between the transmitters and receivers. Further, as the frequency of the EM wave is reduced, their calculated results diverge from the true solution as the physical reality departs from the short-wavelength asymptotic assumptions underlying geometrical optics and ray tracing. Finally, for techniques such as phase screen or Rytov approximations, the calculated results are only valid for weak fluctuations of the ionosphere.

The Finite-Difference Time-Domain (FDTD) method [5,6] is a robust computational EM technique that has been applied to problems across the EM spectrum, from low-frequency geophysical problems below 1 Hz and up into the optical frequency range [6]. The advantages of FDTD for

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Earth-ionosphere wave propagation problems include [7,8]:

- As a grid-based method, the 3-D spatial material variations of the ionosphere composition, topography/bathymetry, lithosphere composition, geomagnetic field, targets, and antennas, etc., may be accommodated. Figure 1 for example, shows FDTD-calculated global EM propagation in the Earth-ionosphere waveguide below 1 kHz that includes details of the Earth's topography, bathymetry, oceans, and an (isotropic) conductivity layering in the ionosphere, which is sufficient for propagation below 1 kHz.
- The complex shielding, scattering and diffraction of EM wave may be calculated in a straightforward manner.
- Any number of simultaneous sources may be accommodated (antennas, plane waves, lightning, ionospheric currents, etc.).
- Any number of observation points may be accommodated, and movies may be created of the time-marching propagating waves.
- As a time-domain method, FDTD can model arbitrary time-varying source waveforms, movement of objects, and time variations in the ionosphere.
- Results may be obtained over a large spectral bandwidth via a discrete Fourier transform.
- A fully 3-D magnetized ionospheric plasma FDTD algorithm may be used to calculate all important ionospheric effects on signals, including absorption, refraction, phase and group delay, frequency shift, polarization, and Faraday rotation.

The downside of being able to accommodate all of the above details and physics, is that the FDTD model may quickly become very memory- and time-intensive, and thus, require significant supercomputing resources. This makes real-time calculations difficult or sometimes even impossible to obtain. Further, if the EM frequency is high enough (and the required grid resolution low enough), the required grid size may become computational infeasible, especially for long propagation paths.

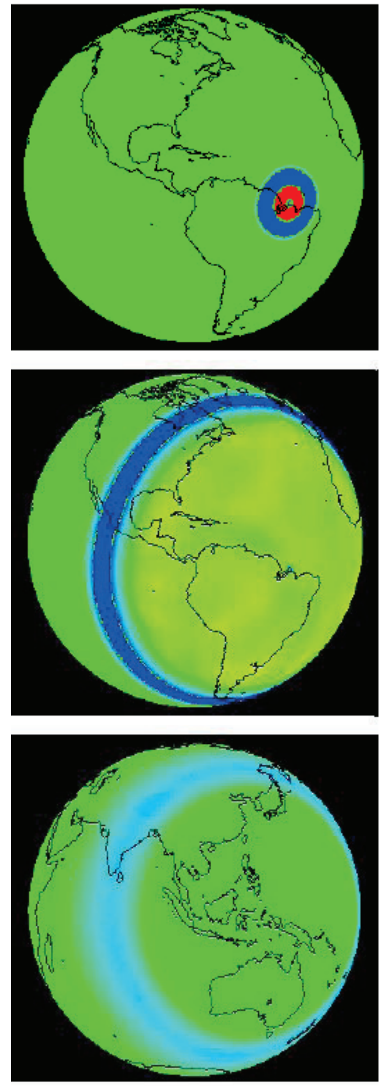

Fig. 1. Snapshot visualizations of round-the-world EM propagation below 1 kHz as calculated by a 3-D FDTD model, including details of the Earth's topography, oceans, and isotropic ionosphere (figure courtesy of [7]).

Although supercomputing capabilities continue to improve, efficient FDTD algorithms are needed to make EM wave propagation modeling in the ionosphere feasible and manageable. Section II provides an overview of the current state of the art for trans-ionospheric EM wave propagation. Section III then describes a new, efficient, 3-D

FDTD magnetized ionospheric plasma model [9] that may be used to greatly advance the current state of the art. Next, Section IV describes a new capability: Stochastic FDTD (S-FDTD) [10,11] magnetized ionospheric plasma modeling [12], which can yield both average as well as variance electric and magnetic fields due to variances and uncertainties in the ionosphere composition. Section V gives an overview of the input parameters that may be used to populate the Earth-ionosphere models. The paper then concludes with a discussion of application possibilities of these models.

II. CURRENT STATE OF THE ART

In 1837, W. R. Hamilton introduced a system of differential equations describing ray paths through general anisotropic media [13]. In 1954, J. Haselgrove proposed that Hamilton's equations were suitable for numerical integration on electronic computers and could provide a means of calculating ray paths in the ionosphere [14]. In 1960, Haselgrove and Haselgrove implemented such a ray-tracing program to calculate "twisted ray paths" through a model ionosphere using Cartesian coordinates [15,1].

In 1975, M. Jones and J. J. Stephenson generated "an accurate, versatile FORTRAN computer program for tracing rays through an anisotropic medium whose index of refraction varies continuously in three dimensions" [16]. This model and variations of it are still in use today, and have been applied to such applications as over-the-horizon radar [1]. Additionally, many other related techniques have now been generated especially for higher frequency scintillation studies, including the phase screen [3] or Rytov approximation, parabolic equation method [2], and even hybrid methods, such as combining the complex phase method and the technique of a random screen [4].

These techniques, however, are only valid under certain conditions. The complex phase method, for example, is only valid for EM wave propagation above 1 GHz. The phase screen or Rytov approximation is only valid for weak fluctuations of the ionosphere. And for all of these methods involving ray tracing, as the frequency of the EM wave is reduced and its wavelength increases, the calculated results diverge from the true solution as the physical reality departs from the short-wavelength asymptotic assumptions underlying geometrical optics and ray tracing [17].

Ray tracing has been traditionally employed for ionospheric propagation because it is computationally inexpensive; however, it is:

- Incapable of taking into account the variable terrain and structural material properties of and between the transmitters and receivers.
- Restrictive, in that particular methodologies of implementing the ray tracing are limited to certain frequency ranges, and its accuracy depends on the plasma properties.
- It provides solutions at only individual frequencies (steady-state solutions may be obtained; pulses cannot be studied).

An alternative to ray tracing is full-vector Maxwell's equations FDTD modeling, which is not limited by the above issues.

FDTD plasma models have been developed by a number of groups [e.g., 18-20]. However, all of these models require large amounts of computer memory, require very small time steps linked to the plasma parameters rather than the Courant limit, or produce nonphysically spurious electrostatic waves (of numerical origin) due to the spatially non-collocated status of electric fields and current densities, resulting in late-time instabilities [17]. Section III describes an FDTD plasma method [12] that does not suffer from these drawbacks.

To study the performance capability of an example FDTD plasma algorithm, FDTD plasma model results have previously been compared to ray-tracing results for the application of reducing the radar cross-section of targets [21]. Although Chaudhury and Chaturvedi limited their study to unmagnetized, collisional cold plasmas, they conclude that FDTD is more accurate and less restrictive than ray tracing, at the cost of being more computationally demanding. For example, they determine that ray tracing only yields accurate results in their study when both the density scale length is long compared to the free-space wavelength of the incident wave, and when the conduction current is small as compared to the displacement current in the medium. Additionally, ray tracing provides solutions at only individual frequencies (i.e., for sinusoidal steady-state signals, not for pulses).

III. MAGNETIZED IONOSPHERIC PLASMA ALGORITHM

The new FDTD plasma modeling methodology is analogous for each of the electrons, positive ions, and negative ions, so due to space constraints, only electrons will be considered here. In the new formulation [12], the coupled Maxwell's equations-Lorentz equation of motion plasma model may be solved using an algorithm originally used by Borris [22] to calculate the velocity of particles in Particle-In-Cell (PIC) plasma models [23]. PIC codes track trajectories of particles or groups of particles ("super-particles") and solve for electrodynamic fields. By using the Borris approach, the resulting FDTD plasma model is stable while also reducing the memory requirements and the execution time compared to all previous FDTD plasma formulations [12,17].

A. Collisional plasma algorithm

The plasma may be considered collisionless or collisional. This section will consider the more general collisional cases. Under the cold plasma condition and by assuming a known electron density, the momentum equation can be simplified as follows [18]:

$$\frac{\partial \overrightarrow{J_e}}{\partial t} + \nu \overrightarrow{J_e} = \epsilon_0 \omega_{pe}^2 \vec{E} + \overrightarrow{\omega_{ce}} \times \overrightarrow{J_e}, \tag{1}$$

where $\overrightarrow{\mathrm{J_e}}$ is electric current due to electrons, ν is the collision frequency, ϵ_0 is the electric permittivity, ω_{pe} is the electron plasma frequency, and $\overrightarrow{\omega_{ce}}$ is the electron gyro-frequency.

The difficulty in solving equation (1) in the collisional regime is that the current density vector is needed at time step $n+\frac{1}{2}$, which is not yet known. In order to solve this issue, a two-step method known as the predictor-corrector method is employed. In the first (predictor) step, the current vector at $n-\frac{1}{2}$ is used to predict the current density at $n+\frac{1}{2}$. In the second (corrector) step, the predicted current density vector from the first step is used and all the equations are solved again. A second, new current density vector is found at $n+\frac{1}{2}$, that is known as the corrector current density vector. The average of the predicted current density vector and the corrector current density vector at $n+\frac{1}{2}$ is used for current density vector at $n+\frac{1}{2}$. The predictor-corrector method is second order accurate [24,25].

Equation (1) in discrete form in the predictor step [12,22] is as follows:

$$\frac{\vec{J}_{e,p}^{\,n+\frac{1}{2}} - \vec{J}_{e}^{\,n-\frac{1}{2}}}{\Delta t} + \nu \vec{J}_{e}^{\,n-\frac{1}{2}} = +\epsilon_0 \omega_{pe}^2 \vec{E}^n 0 - \omega_{C_e}^2 \times \left(\frac{\vec{J}_{e,p}^{\,n+\frac{1}{2}} + \vec{J}_{e}^{\,n-\frac{1}{2}}}{2} \right). \tag{2}$$

In equation (2), it appears that the current density components should be collocated with the electric field component. However, in the time domain, the current densities are solved out of sync with the electric fields, which are solved at each integer time step; i.e., (n). Instead, the current densities are solved at the same time step as the *H*-fields (or at each half time step); i.e., $(n+\frac{1}{2})$. In order to simplify equation (2), the *E*-field should be incorporated into the current vector term. We define two auxiliary current density components [12,22] as follows:

$$\vec{J}^{+} = \vec{J}_{e,p}^{\,n+\frac{1}{2}} - \frac{\Delta t \epsilon_0 \omega_{pe}^2 \vec{E}^n}{2} + \frac{\Delta t \nu \vec{J}_{e}^{\,n-\frac{1}{2}}}{2}, \tag{3}$$

$$\vec{J}^{-} = \vec{J}_{e}^{\,n-\frac{1}{2}} + \frac{\Delta t \epsilon_0 \omega_{pe}^2 \vec{E}^n}{2} - \frac{\Delta t \nu \vec{J}_{e}^{\,n-\frac{1}{2}}}{2}. \tag{4}$$

The cross product does not change the energy; therefore, $|J^{+}| = |J^{-}|$. However, the direction of the vector is changed. Figure 2 demonstrates the rotation of the current density vector around $\overrightarrow{\omega_{C_e}}$ that is for simplicity (only for the figure) assumed to be perpendicular to the current density components. The direction of the $\overrightarrow{\omega_{C_e}}$ and the *B*-field is out of the paper.

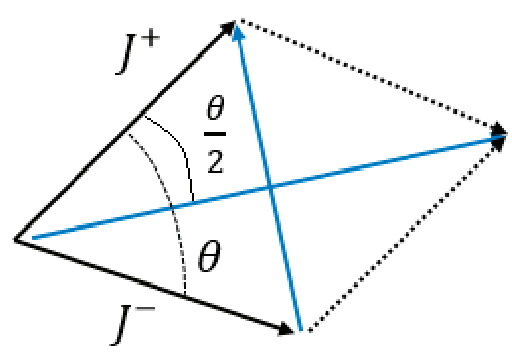

Fig. 2. Rotation of the current vector around $\overrightarrow{\omega_{C_e}}$. Figure adapted from [23].

From Fig. 2, the angle of rotation is:

$$\frac{\theta}{2} = \tan^{-1}\frac{\vec{J}^{+}-\vec{J}^{-}}{\vec{J}^{+}+\vec{J}^{-}} = \tan^{-1}\frac{|\overrightarrow{\omega_{C_e}}|\Delta t}{2}. \quad (5)$$

The sampling frequency should be twice the electron gyro-frequency to accurately model it. Therefore, $\Delta t < \frac{\pi}{\omega_{ce}}$, which means $\theta \leq 115°$. For smaller angles the results are more precise. The $\vec{J}^{+}$ can be found in four steps as follows [12,22,23]:

$$\vec{J}_0 = \vec{J}^{-} \times \vec{t}, \quad (6.1)$$

$$J_1 = \vec{J}^{-} + \vec{J}_0, \quad (6.2)$$

$$\vec{J}_2 = \vec{J}_1 \times \vec{s}, \quad (6.3)$$

$$\vec{J}^{+} = \vec{J}^{-} + \vec{J}_2, \quad (6.4)$$

where $\vec{t} = -\frac{\overrightarrow{\omega_{C_e}}}{|\overrightarrow{\omega_{C_e}}|}\tan(\frac{\theta}{2})$, and $\vec{s} = \frac{\overrightarrow{\omega_{C_e}}}{|\overrightarrow{\omega_{C_e}}|}\sin(\theta)$.

As part of the predictor-corrector method, equation (1) is discretized in the corrector step as follows [12,22]:

$$\frac{\vec{J}_{e,c}^{\,n+\frac{1}{2}} - \vec{J}_{e}^{\,n-\frac{1}{2}}}{\Delta t} + \nu\vec{J}_{e,p}^{\,n+\frac{1}{2}} = +\epsilon_0\omega_{pe}^2\vec{E}^n - \omega_{C_e}^2 \times \left(\frac{\vec{J}_{e,c}^{\,n+\frac{1}{2}} + \vec{J}_{e}^{\,n-\frac{1}{2}}}{2}\right). \quad (7)$$

The auxiliary current density vectors are then defined as [12,22]:

$$\vec{J}^{+} = \vec{J}_{e,c}^{\,n+\frac{1}{2}} - \frac{\Delta t\epsilon_0\omega_{pe}^2\vec{E}^n}{2} + \frac{\Delta t\nu\vec{J}_{e,p}^{\,n+\frac{1}{2}}}{2}, \quad (8)$$

$$\vec{J}^{-} = \vec{J}_{e}^{\,n-\frac{1}{2}} + \frac{\Delta t\epsilon_0\omega_{pe}^2\vec{E}^n}{2} - \frac{\Delta t\nu\vec{J}_{e,p}^{\,n+\frac{1}{2}}}{2}. \quad (9)$$

The final current vector is [12,22]:

$$\vec{J}_{e}^{\,n+\frac{1}{2}} = \frac{\vec{J}_{e,p}^{\,n+\frac{1}{2}} + \vec{J}_{e,c}^{\,n+\frac{1}{2}}}{2}. \quad (10)$$

The maximum allowed time step that may be used depends on the electron gyro frequency; i.e., $\Delta t < \frac{\pi}{\omega_{ce}}$.

Several validation tests are performed in [Samimi and Simpson, submitted] to demonstrate the accuracy and capability of the newly developed FDTD plasma model. Section III (B) below provides an example validation, and Section III (C) summarizes the performance of this new model.

B. Example validation

As an example validation, the propagation of an electromagnetic wave inside a small plasma spherical waveguide is investigated. This simulation case is chosen so that propagation over a short distance may be modeled and compared to theory. It serves as a high-resolution validation of the global FDTD plasma model of [26,12]. Additional validation cases are provided in [12].

The spherical waveguide has an internal radius 2.673 m and external radius 3.6978 m. A magnetic field is considered in the south-north direction and its strength is $B_0 = 0.06\, T$. The electron density is $n_e = 10^{18}\, 1/m^3$. The source of the electromagnetic wave is located at $30°\, S$ and propagation toward the equator is examined. The source creates a linearly polarized electromagnetic plane wave polarized in the radial (r)-direction and having a Gaussian time-waveform according to:

$$\mathrm{E}_{source} = \exp\left(-\frac{(\mathrm{t}-50\Delta \mathrm{t})^2}{2(7\Delta \mathrm{t})^2}\right). \quad (11)$$

This pulse is expected to excite the R-wave and L-wave as well as low frequency whistler mode. The whistler mode is part of the R-wave dispersion relation that can propagate at frequencies less than the electron gyro-frequency.

Figure 3 shows the time domain electric field in the r-direction; i.e., $\mathrm{E_r(t)}$, 40 cells (approximately 40 mm) from the source. The low frequency whistler mode arrives at the observation point at around 1.2 ps. Figure 4 shows the power spectrum of the electric field corresponding to the time-waveform of Fig. 3. The L-wave cutoff frequency, ω_L, the R-wave cutoff frequency, ω_R, and the whistler mode with frequency band less than the electron cyclotron frequency ($< \omega_\mathrm{ce}$) are apparent in the figure. These results are also in very good agreement with plasma theory and the simulation results of the previous anisotropic model [26].

Note, that in this validation test, the time step value for solving Maxwell's equations is chosen according to the Courant stability condition and is $\Delta \mathrm{t} = 1.5$ ps. This time step value corresponds to a rotation angle $\theta = 0.9°$ that yields a numerical electron gyro-frequency error of less than 0.5%. Therefore, there is no need to use a different time step for solving the current equation compared to

Maxwell's equations.

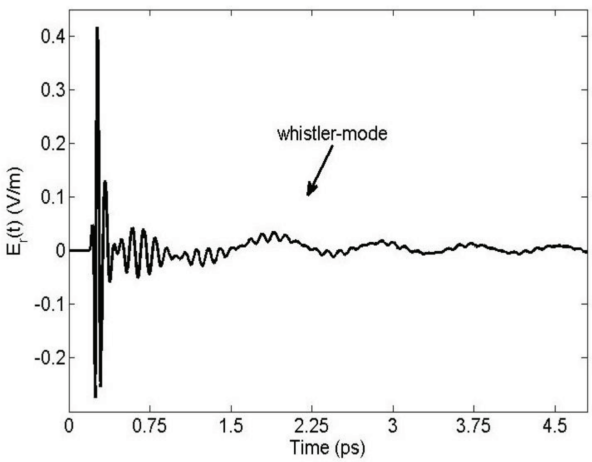

Fig. 3. Time-domain electric-field waveform in the r-direction recorded ~40 mm from the source along the magnetic field (figure courtesy of [12]).

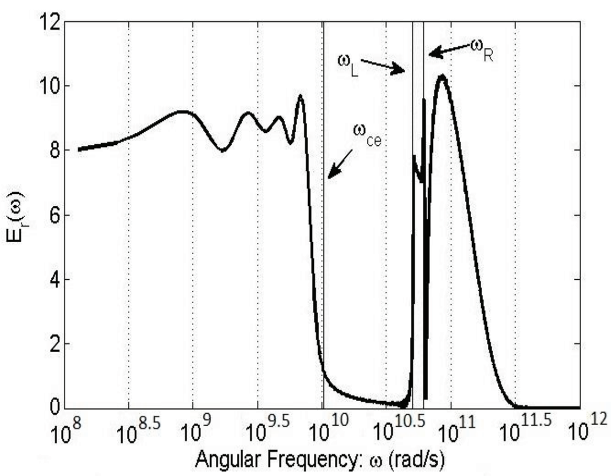

Fig. 4. Power spectrum of the electric field in the r-direction recorded ~40 mm from the source along the magnetic field line (figure courtesy of [12]).

C. Summary of performance

The advantages of the new FDTD plasma model [12] over the previous formulation of [17] are as follows:

- It permits the use of two different time steps for solving the current equation vs. the Maxwell's equations. The previous anisotropic model did include this capability, and so for some cases the time-step requirements of the current density solutions could drastically slow down the solutions to the Maxwell's equations. As such, obtaining solutions for cases involving high collision frequencies was nearly impossible due to the necessary long computational time.
- It is faster than the previous model. Depending upon the size of the time step needed to solve the current equation, the new algorithm is more than 50 percent faster than the previous version.
- Implementation of the algorithm is much simpler and no matrix equation must be solved.
- The memory requirements are drastically less than for the previous formulation (3 additional real numbers are stored per cell relative to traditional FDTD compared to 9 additional real numbers stored per cell as for the previous plasma formulation; also, it does not require storage or re-calculation of a coefficient matrices of size at least 6x6 at every grid cell).

The only disadvantage of the new algorithm is that for simulating wave propagation in dense plasma, the stability condition can be smaller than Courant limit. The plasma frequency puts an additional restriction on the maximum allowable time step value. Therefore, either the Courant condition or a $dt < \frac{0.87}{\omega_{pe}}$, whichever is smaller, should be chosen for the time step for Maxwell's equations.

IV. STOCHASTIC FDTD

A second recent development that has advanced time-domain modeling of ionospheric propagation is a new stochastic FDTD plasma model that solves for mean as well as variance electromagnetic fields due to uncertainties or variances in the ionosphere composition. The variability of the ionosphere renders many propagation problems too complex to be solved using a deterministic formulation. The structure of the ionosphere can depend not only on the altitude, time of day, and season, but also on the latitude, longitude, sun spot cycle, and occurrence of space weather events. A useful approach to such a highly complex problem is to consider it as a random medium problem.

Numerical EM techniques, however, typically use only average (mean) values of the constitutive parameters of the materials and then solve for expected (mean) electric and magnetic fields. The Monte Carlo method is a well-established and widely-used brute force technique for evaluating random medium problems via multiple realizations

[10]. Depending on the nature of the statistical correlation, a random medium problem may require tens or hundreds of thousands of realizations. This yields an extremely inefficient brute force approach, particularly for 2-D and 3-D problems, and therefore is rarely used in EM modeling.

Stochastic FDTD (S-FDTD) is an efficient formulation that runs the ensemble averages in a single realization scheme [10,11]. S-FDTD was recently extended to EM wave propagation in the ionosphere by extending the stochastic variables to both Maxwell's equations and the Lorentz equation of motion [9]. The electric fields, magnetic fields, current densities, electron/ion densities and collision frequencies all are treated as random variables with their own statistical variation. The resulting mean and variance calculations of the EM fields and current densities provides new capabilities; for example, the ability to determine the confidence level that a communications/remote sensing/radar system will operate as expected under abnormal ionospheric conditions. It may also be useful in a wide variety of geophysical studies.

The advantage of S-FDTD is that it requires only about twice as much computer simulation time and memory as a traditional FDTD simulation regardless of the number of random variables. On the other hand, its limitation is that it can only bound the field variances according to a best estimate approximation for the cross correlation coefficients.

In [9], an S-FDTD method is developed for the previous (less efficient) magnetized plasma algorithm of [17]. Recently, a more efficient magnetized plasma model was developed [12] as presented in Section III of this paper. In the remainder of this section, general guidelines are provided for extending the S-FDTD approach to the more efficient magnetized plasma model of Section III and [12]. The general approach is analogous to that of [9].

A. Mean field equations

Using the Delta method [27], the average (or expected) EM fields and current density values may be found by solving Maxwell's equations and the current equation while using mean (average) values of the variables [10]. For the S-FDTD magnetized cold plasma model, the equations for the mean values of the EM fields and current densities are of the same form as for those of the regular 3-D FDTD magnetized cold plasma model. Thus, the mean EM field and current density values are found by using the mean plasma frequency of ω_{Pe}, or equivalently, the mean of electron density n_e.

B. Variance field equations

The variance fields may also be derived by using the delta method and the statistical values. When solving only Maxwell's equations, the variance field equations may be solved separately from the mean field equations no matter the dimensionality of the problem [10]. However, in the 3-D magnetized cold plasma model, the momentum equation (1) is coupled to Maxwell equations, which leads to a complicated but linear system. As a result, the electric field and current density variances must be computed simultaneously. When variance equations are derived, covariances are needed for the E, H fields and current density J_e in both time and space. Equation (1) also relates the current density to the collision frequency and the electric field to the plasma frequency of the ionosphere, resulting in additional covariance terms of between the current density and collision frequency, and the electric field and plasma frequency. For S-FDTD method, a critical step is to approximate these correlation coefficients, which controls the accuracy of the algorithm.

Figure 5 shows a diagram of the iteration process for each time step of the S-FDTD method. What is changed from regular FDTD updating is the addition of the calculation of the variances after the mean values are obtained. Therefore, the running time as well as the memory required for S-FDTD is roughly double that needed for traditional FDTD (and double that for the regular FDTD plasma model). Also, since both the mean fields and their variances behave like waves, both require boundary conditions. Thus, an absorbing boundary condition is needed for the E, H, and J_e mean values as well as for their variances. In [9], Mur's boundary conditions are used because that boundary condition was found to provide good absorption regardless of the magnetic field direction [28].

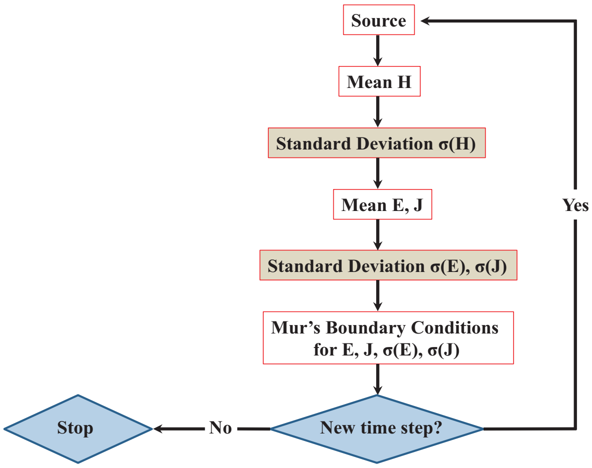

Fig. 5. S-FDTD flow chart (figure adapted from [10]).

V. INPUT TO THE FDTD MODELS

Since the FDTD model may account for highly detailed structures and materials, it is useful to populate the FDTD grid with realistic data. The Earth's topography and bathymetry data may be obtained; for example, from the National Oceanic and Atmospheric Administration (NOAA) National Geophysical Data Center (NGDC). The Earth's magnetic field data and its direction and amplitude variation with position may be obtained from the International Geomagnetic Reference Field (IGRF).

For an isotropic conductivity profile ionosphere to be used in lower frequency EM propagation models, relatively simple profiles based on measurements and analytical calculations may be used, such as an exponential conductivity profile [29] or a knee profile [30]. To model an anisotropic magnetized plasma ionosphere to be used in higher frequency EM propagation models, electron and ion densities and collision frequencies and their variation with time and position may be obtained from the International Reference Ionosphere (IRI) and other sources. IRI has recently been expanded to include stochastic information about the ionosphere composition (e.g., [31]).

VI. CONCLUSION

This paper provided an overview of two recent advances in FDTD EM wave propagation modeling in the ionosphere:

(1) A new, efficient 3-D magnetized ionospheric plasma model.
(2) A stochastic FDTD model of ionospheric plasma.

The combination of these models provides the capability to model high frequency EM wave propagation over longer distances than previously possible, while also solving for not only mean but also variance electric and magnetic fields due to uncertainties or variances in the ionosphere. Applications of these models range from remote sensing to communications and space weather.

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Bach T. Nguyen was born in Hanoi, Vietnam. He received his B.Eng. and M.Eng. degrees in Electrical Engineering from National Defense Academy of Japan, Yokosuka, Kanagawa, Japan, in 2007 and 2009, respectively. From 2010 to 2012, he was with the Vietnam Military Academy of Science and Technology, Hanoi, Vietnam. Currently, he is pursuing his Ph.D. degree in the Electrical and Computer Engineering Department, University of Utah, Salt Lake City, UT. His research interests include computational electromagnetics, RF/Microwave technology, RF-IC, and liquid crystal. His current research focuses on stochastic FDTD simulation of electromagnetic wave propagation in the ionosphere.

Alireza Samimi received his B.S. degree in Electrical Engineering from Shiraz University, Shiraz, Iran in 2005, his M.S. degree in Electrical Engineering from University of Tabriz, Tabriz, Iran, in 2008, and his Ph.D. degree in Electrical Engineering from Virginia Tech in 2013.His research interests include physics of the upper atmosphere, Finite-Difference Time-Domain (FDTD) solution of Maxwell's equations and its applications in simulating wave propagation in the Earth-Ionosphere-Magnetosphere system, active modification of the ionosphere and particle-in-cell computational modeling of plasma instabilities.

He is currently working as a Postdoctoral Fellow in the Department of Electrical and Computer Engineering, University of Utah, Salt Lake City. He participated in two experimental research campaigns at High Frequency Active Auroral Research Program (HAARP) facilities to study generation mechanism of the narrow-band stimulated electromagnetic emission spectral features excited during ionospheric heating near second electron gyro-harmonic. Samimi has been a member of American Geophysical Union (AGU) since 2010 and IEEE since 2011.

Jamesina J. Simpson received her B.S. and Ph.D. degrees in Electrical Engineering from Northwestern University, Evanston, IL, in 2003 and 2007, respectively. In 2007, she joined the Electrical and Computer Engineering Department, University of New Mexico, Albuquerque, as a tenure-track Assistant Professor. As of July 2012, she is an Associate Professor in the Electrical and Computer Engineering Department, University of Utah, Salt Lake City. Her research lab encompasses the application of the full-vector Maxwell's equations Finite-Difference Time-Domain (FDTD) method to a wide variety of scientific and engineering applications across the electromagnetic spectrum.

Simpson is a recipient of the National Science Foundation (NSF) Graduate Research Fellowship and the IEEE AP-S and MTT-S Graduate Research Awards. In 2010, she received an NSF CAREER award, and in 2012 she received the Donald G. Dudley, Jr. Undergraduate Teaching Award of the IEEE AP-S. From 2010-2014, Simpson was an Associate Editor of IEEE Transactions on Antennas and Propagation.