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ALGEBRA
SIMULTANEOUS EQUATIONS
Example 1

a) Solve following eqns for V_1 and V_2 in terms of R's:

$$V_1\left(\frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_3}\right)-V_2\frac{1}{R_3}=10$$

$$\frac{V_1-V_2}{R_3}=\frac{V_2}{R_4}$$

soln: Rewrite 2nd eqn as $\frac{V_1}{R_3}=V_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$

Use parallel R notation $R_1\|R_2=\frac{R_1R_2}{R_1+R_2}=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$

Our two eqns become:

$$\frac{V_1}{R_1\|R_2\|R_3}-\frac{V_2}{R_3}=10$$

$$\frac{V_1}{R_3}-\frac{V_2}{R_3\|R_4}=0$$

Now use 2nd eqn to write V_2 in terms of V_1:

$$V_2=V_1\frac{R_3\|R_4}{R_3}=V_1\frac{R_3R_4}{(R_3+R_4)\cdot R_3}=V_1\frac{R_4}{R_3+R_4}$$

Now substitute this into first eqn:

$$\frac{V_1}{R_1\|R_2\|R_3}-\frac{V_1}{R_3}\frac{R_4}{R_3+R_4}=10\ A$$

$$V_1=10\Big/\left(\frac{1}{R_1\|R_2\|R_3}-\frac{R_4}{R_3}\frac{1}{R_3+R_4}\right)=10\Big/\left(\frac{1}{R_1\|R_2\|R_3}-\frac{R_3\|R_4}{R_3^2}\right)$$

$$V_2=V_1\frac{R_3\|R_4}{R_3}=10\Big/\frac{R_3}{R_3\|R_4}\left(\frac{1}{R_1\|R_2\|R_3}-\frac{R_4}{R_3}\frac{1}{R_3+R_4}\right)$$

$$V_2=10\Big/\left(\frac{R_3+R_4}{R_4}\frac{1}{R_1\|R_2\|R_3}-\frac{R_3\|R_4}{R_3(R_3\|R_4)}\right)$$

$$V_2=10\Big/\left(\frac{R_3/R_4\|R_3}{R_1\|R_2\|R_3}-\frac{1}{R_3}\right)=10R_3\Big/\left(\frac{R_3^2/R_4\|R_3}{R_1\|R_2\|R_3}-1\right)$$

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b) Make at least two consistency checks on answer to (a).

Units: $V_1 = A \Big/ \left(\frac{1}{\Omega} - \frac{\Omega}{\Omega^2} \frac{1}{\Omega} \right) = A \cdot \Omega = V$ ✓

$$V_2 = A\Omega \Big/ \left(\frac{\Omega^2/\Omega}{\Omega} - 1 \right) = A \cdot \Omega = V$$ ✓

When $R_3 \to \infty$ 2nd eqn gives $V_2 = 0$, 1st eqn gives

$$V_1 = 10A \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

In our V_1 eqn with $R_3 \to \infty$ we get $R_1 \| R_2 \| R_3 = R_1 \| R_2$,

$$\frac{R_4}{R_3} \frac{1}{R_3 + R_4} = \frac{R_4}{\infty (\infty + R_4)} = \frac{R_4}{\infty} = 0$$

$$\therefore V_1 = \frac{10}{V_{R_1 \| R_2}} = 10 \cdot R_1 \| R_2 = 10 \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$ ✓

$$V_2 = 10 \Big/ \left(\frac{\infty \, / \, R_4}{R_1 \| R_2} - \frac{1}{\infty} \right) = 10 \cdot \frac{R_1 \| R_2}{\infty} = 0$$ ✓

Make another consistency check.

$R_4 \to \infty$ gives $V_1 = V_2$ from 2nd eqn. and $R_3 \| R_4 = R_3$.

Our eqns: $$V_1 = 10 \Big/ \left(\frac{1}{R_1 \| R_2 \| R_3} - \frac{R_3}{R_3^2} \right) = 10 \Big/ \left(\frac{1}{R_1 \| R_2 \| R_3} - \frac{1}{R_3} \right)$$

$$V_2 = 10 \Big/ \left(\frac{R_3 / R_3}{R_1 \| R_2 \| R_3} - \frac{1}{R_3} \right) = 10 \Big/ \left(\frac{1}{R_1 \| R_2 \| R_3} - \frac{1}{R_3} \right)$$ ✓

One more check. $R_1 = \infty$ $R_2 = \infty$ $R_3 = R_4$ Then $V_1 = 2V_2$.

$V_2 = \frac{V_1}{2}$, $V_1 - V_2 = 10 R_3$, $V_1 - \frac{V_1}{2} = 10 R_3$, $V_1 = 2 \cdot 10 R_3$.

Our eqns: $$V_1 = 10 \Big/ \left(\frac{1}{R_3} - \frac{R_3/2}{R_3^2} \right) = 10 \Big/ \frac{1}{2R_3} = 2 \cdot 10 R_3$$ ✓

$$V_2 = 10 \Big/ \left(\frac{R_3/(R_3/2)}{R_3} - \frac{1}{R_3} \right) = 10 \Big/ 1/R_3 = 10 R_3 = \frac{V_1}{2}$$ ✓