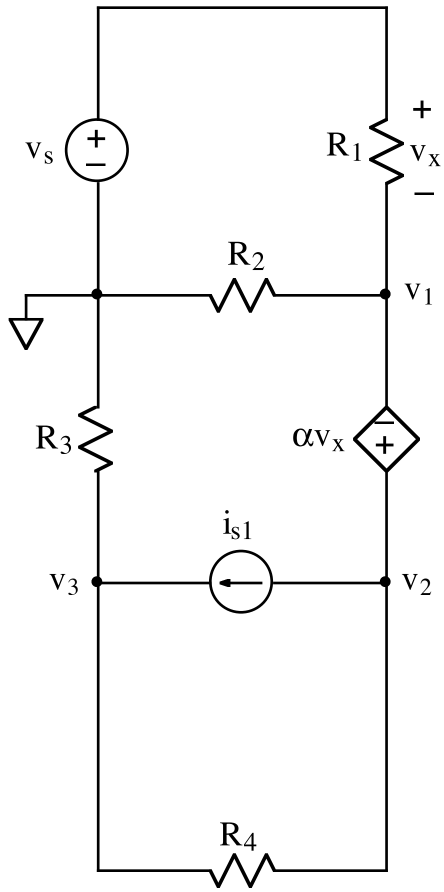
Ex:



(1) $$\frac{v_1 - v_s}{R_1} + \frac{v_1}{R_2} + i_s + \frac{v_2 - v_3}{R_4} = 0A$$

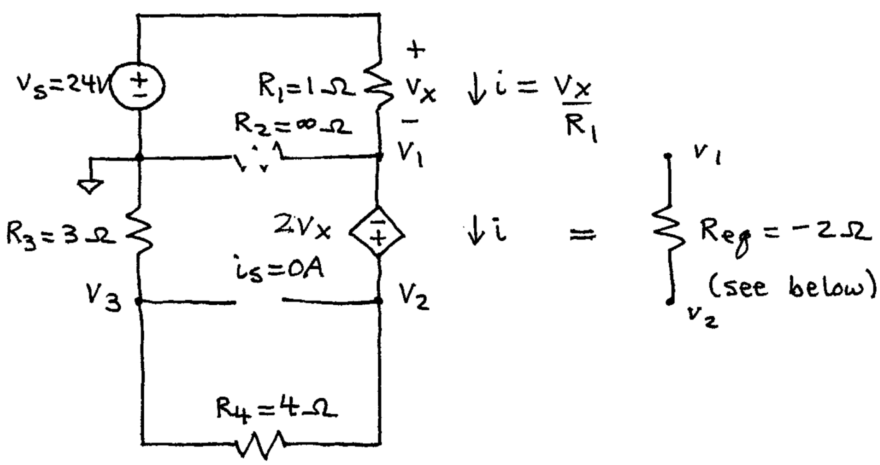
(2) $$v_1 + \alpha(v_s - v_1) = v_2$$

(3) $$\frac{v_3}{R_3} - i_s + \frac{v_3 - v_2}{R_4} = 0A$$

Make a consistency check on the above node-voltage equations by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and for your consistency check, and show that the equations are satisfied for these values. (In other words, plug the values into the equations and show that the left side and the right side of each equation are equal.)

Sol'n: Many consistency checks are possible. Here, we consider a check in which the dependent source remains on.

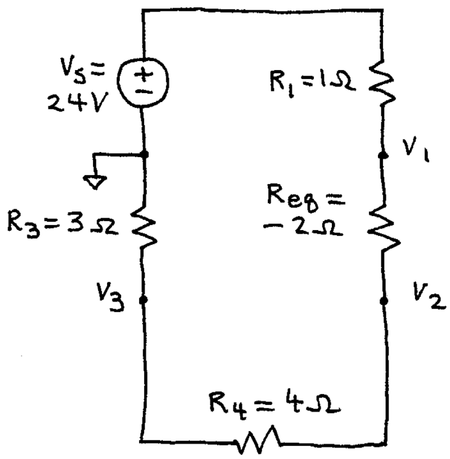
Let $v_s = 24\text{V}$
$R_1 = 1\,\Omega$
$R_2 = \infty\,\Omega$ (open circuit)
$R_3 = 3\,\Omega$
$R_4 = 4\,\Omega$
$\alpha = 2$
$i_s = 0\text{A}$ (off = open circuit)



We observe that current $i = v_x/R_1$ flows thru R_1, creating v_x, and thru the dependent source, creating $-\alpha v_x$. Thus, we may replace the dependent source with an equivalent R:

$$R_{eq} = \frac{v}{i} = \frac{-\alpha v_x}{i} = \frac{-2\,v_x}{\frac{v_x}{R_1 = 1\,\Omega}} = -2\,\Omega$$

Using R_{eq} in place of αv_x gives the follow circuit:



Now we can use v-divider eq'ns to find v_1, v_2, and v_3:

$$v_3 = v_s \cdot \frac{R_3}{R_1 + R_{eq} + R_4 + R_3} = 24V \cdot \frac{3\Omega}{1-2+4+3\ \Omega}$$

$$v_3 = 24V \cdot \frac{3\Omega}{6\Omega} = 12V$$

$$v_2 = v_s \frac{R_3 + R_4}{R_1 + R_{eq} + R_4 + R_3} = 24V \cdot \frac{3+4\ \Omega}{6\Omega}$$

$$v_2 = 28V$$

$$v_1 = v_s \frac{R_{eq} + R_3 + R_4}{R_1 + R_{eq} + R_4 + R_3} = 24V\left(\frac{-2+3+4}{6\Omega}\right)\Omega$$

$$v_1 = 20V$$

We plug all our numbers into the eq'ns given in the problem to verify that equality holds.

(1) $$\frac{20V - 24V}{1\Omega} + \frac{20V}{\infty\Omega} + 0A + \frac{28V - 12V}{4\Omega}$$

$$= \frac{-4V}{1\Omega} + 0A + 0A + \frac{16V}{4\Omega} = -4A + 4A = 0A \checkmark$$

(2) $$20V + 2(24V - 20V) = 20V + 2(4V) = 28V$$

$$= V_2 \checkmark$$

(3) $$\frac{12V}{3\Omega} - 0A + \frac{12V - 28V}{4\Omega}$$

$$= 4A - 0A + \frac{-16V}{4\Omega} = 4A - 4A = 0A \checkmark$$