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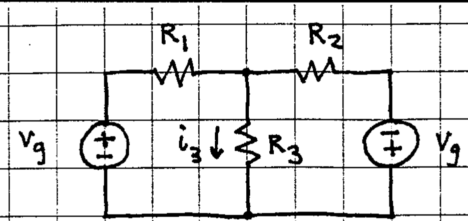
ex:



i) Apply Kirchhoff's laws to find an expression for i_3.

ii) Make at least 2 consistency checks on answer.

iii) Given $v_g = 1\,V$, $R_2 = 100\,\Omega$, and $R_3 = 10\,\Omega$, find R_1 so that $i_3 = 0$.

iv) For $v_g = 1\,V$, $R_3 = 10\,\Omega$, and R_1 = value found in part iii, find i_3 when $R_2 = 200\,\Omega$ and when $R_2 = 50\,\Omega$.

ans: i) $i_3 = v_g \dfrac{R_2 - R_1}{R_1R_2 + R_2R_3 + R_3R_1}$

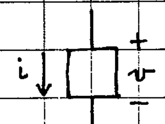
ii) see sol'n, various possibilities

iii) $R_1 = 100\,\Omega$

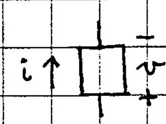
iv) $i_3 = 4.35\text{ mA}$ for $R_2 = 200\,\Omega$ $\qquad$ $i_3 = -7.69\text{ mA}$ for $R_2 = 50\,\Omega$

sol'n: i) We first label voltages and currents. By the passive sign convention, the current measurement arrow must point from the + sign of the voltage measurement toward the − sign of the voltage measurement.

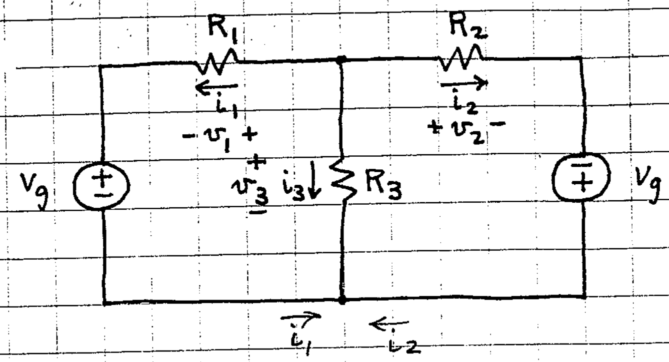
Note: If we follow the passive sign convention, we still have two choices for the direction of the current arrow and the corresponding voltage measurement:



or



In both cases the current measurement arrow points from + to −. The direction of the physical current flow is, of course, unchanged. If, for example, the physical current is flowing down, then we would get a negative value when we solve for i if we use the measurement scheme on the right.



Now we apply Kirchhoff's Laws:

1) Sum of currents flowing out of node = 0.

For the top node at the junction of R_1, R_2, and R_3 we have

$$i_1 + i_2 + i_3 = 0\,A$$

All the terms appear with plus signs because all three current measurement arrows point out from the node. If a current measurement arrow points toward a node, we subtract the current.

Thus, for the bottom node we have

$$-i_1 - i_2 - i_3 = 0\,A$$

We observe that is just $i_1 + i_2 + i_3 = 0A$ multiplied by -1 on both sides. Consequently, this 2nd eq'n doesn't give us any new constraints that help us solve the problem.

moral: At most, we have one less current eq'ns than the number of circuit nodes, (where three or more elements connect).

2) Sum of voltage drops around any loop = 0.

We have a loop on the left and a loop on the right. There is also an outer loop.

The outer loop, it turns out, is redundant. If we add the eq'ns for the smaller loops together, we obtain the eq'n for the outer loop.

This observation is typical. We normally just need the smallest loops. Current sources complicate the picture somewhat, however.

If current sources are present, we avoid using loops that include them. The reason we do so is that the voltage drop across a current source creates a new unknown that we must solve for. Thus, we fail to gain ground in our effort to find as many eq'ns as we have unknowns.

Here, we have no current sources and we can write two loop eq'ns.

Left loop: $v_g + v_1 - v_3 = 0\,V$

Right loop: $v_3 - v_2 + v_g = 0\,V$

Note: I usually travel clockwise around the loop, but the other direction may be used. I treat a voltage drop as positive if I exit the circuit element at the + sign. In other words, I add that v-drop to the total. I subtract that v-drop from the total if I exit the circuit element at the − sign. The Text does the opposite.

Now we apply Ohm's law to obtain 3 more eq'ns:

$v_1 = i_1 R_1 \qquad v_2 = i_2 R_2 \qquad v_3 = i_3 R_3$

Substitute for voltages and solve for i_3.

Our three eqns become

$$i_1 + i_2 + i_3 = 0\text{A} \qquad (1)$$

$$v_g + i_1 R_1 - i_3 R_3 = 0\text{V} \qquad (2)$$

$$i_3 R_3 - i_2 R_2 + v_g = 0\text{V} \qquad (3)$$

Using the first eqn (which has the fewest terms) we solve for i_1:

$$i_1 = -(i_2 + i_3)$$

Substituting into the 2nd and 3rd eqns, we have:

$$v_g + -(i_2 + i_3) R_1 - i_3 R_3 = 0\text{V} \qquad (2)$$

$$i_3 R_3 - i_2 R_2 + v_g = 0\text{V} \qquad (3)$$

We now solve for i_2 using the new third eqn (which has the fewest terms):

$$i_2 = \frac{i_3 R_3 + v_g}{R_2}$$

Substituting into the 2nd eqn, we have:

$$v_g - \left(\frac{i_3 R_3 + v_g}{R_2} + i_3\right) R_1 - i_3 R_3 = 0\text{V}$$

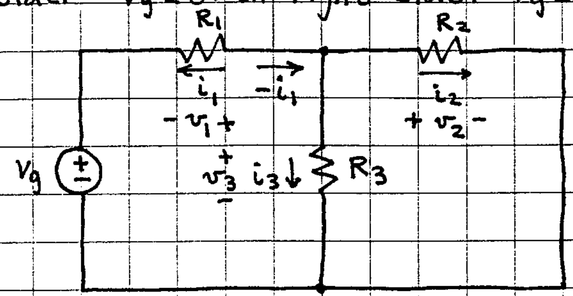
We solve this eqn for i_3:

$$i_3 \left(\frac{R_3}{R_2} R_1 + R_1 + R_3\right) = v_g \left(1 - \frac{R_1}{R_2}\right)$$

$$i_3 = v_g \frac{(R_2 - R_1)}{R_1 R_3 + R_1 R_2 + R_2 R_3} \qquad \text{units consistent} \checkmark$$

(cont) ii) • Consider $V_g = 0$: No energy sources $\Rightarrow i_3 = 0$
Agrees with our formula. ✓

• Consider $V_g = 0V$ on right side: $V_g = 0V \Rightarrow$ short circuit



Total current, $-i_1 = \dfrac{V_g}{R_1 + R_2 \| R_3}$.

By current-divider formula, $i_3 = -i_1 \dfrac{R_2}{R_2 + R_3} = \dfrac{R_2 \| R_3}{R_3}$.

$$\therefore \; i_3 = \frac{V_g}{R_1 + R_2\|R_3} \frac{R_2\|R_3}{R_3} = \frac{V_g}{R_1 + \dfrac{R_2R_3}{R_2+R_3}} \frac{\dfrac{R_2\cancel{R_3}}{R_2+R_3}}{\cancel{R_3}}$$

$$= V_g \frac{R_2}{R_1(R_2+R_3) + R_2R_3}$$

Now a trick: By symmetry, if we set $V_g = 0V$ on the right side (instead of the left) and turn it upside down (as in the original circuit), we must have

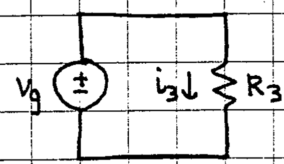
$$i_3 = -V_g \frac{R_1}{R_2(R_1+R_3) + R_1R_3}$$

If we add these two i_3 values together, we get our formula for i_3 from part (i). ✓

This is a preview of the idea of superposition. More on this later on in the course.

• Consider $R_3 = \infty$: $i_3 = 0A$ since open circuit for R_3
Agrees with our formula. ✓

ii (cont) • Consider $R_1 = 0\,\Omega$ and $R_2 = \infty\,\Omega$: Just v_g and R_3



$$i_3 = \frac{v_g}{R_3}$$

Our formula with $R_1 = 0$: $i_3 = \dfrac{v_g R_2}{R_2 R_3} = \dfrac{v_g}{R_3}$

$$\lim_{R_2 \to \infty} \frac{v_g}{R_3} = \frac{v_g}{R_3} \quad \checkmark$$

Note: It actually doesn't matter what R_2 is if $R_1 = 0\,\Omega$. The voltage across R_3 will be v_g, and the current thru R_3 must be v_g/R_3.

iii) If $R_2 = 100\,\Omega$ then $i_3 = 0$ when $R_1 = R_2 = 100\,\Omega$

iv) Given: $R_1 = 100\,\Omega$, $R_3 = 10\,\Omega$, $v_g = 1V$

If $R_2 = 200\,\Omega$, then $i_3 = 1V \dfrac{(200\,\Omega - 100\,\Omega)}{100\,\Omega\, 10\,\Omega + 100\,\Omega\, 200\,\Omega + 200\,\Omega\, 10\,\Omega}$

or $i_3 = 1V \cdot \dfrac{\cancel{100\,\Omega}}{\cancel{100\,\Omega}\,(10\,\Omega + 200\,\Omega + 20\,\Omega)} = \dfrac{1V}{230\,\Omega}$

or $i_3 = 4.35$ mA.

If $R_2 = 50\,\Omega$, then $i_3 = 1V \dfrac{(50\,\Omega - 100\,\Omega)}{100\,\Omega\, 10\,\Omega + 100\,\Omega\, 50\,\Omega + 50\,\Omega\, 10\,\Omega}$

or $i_3 = 1V \cdot \dfrac{(-\cancel{50\,\Omega})}{\cancel{50\,\Omega}\,(20\,\Omega + 100\,\Omega + 10\,\Omega)} = \dfrac{-1V}{130\,\Omega}$

or $i_3 = 7.69$ mA.