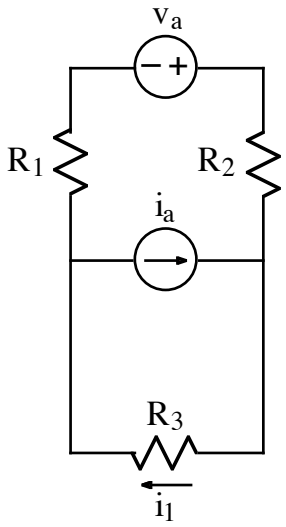
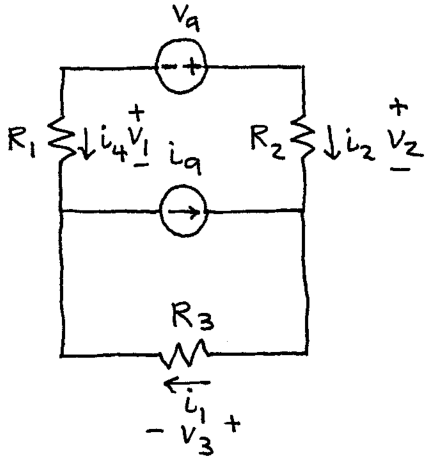
**EX:**




Derive an expression for $i_1$. The expression must not contain more than the circuit parameters $v_a$, $i_a$, $R_1$, $R_2$, and $R_3$.

sol'n: We label i's and v's for R's.




We have a v-loop around the outside.
(other loops would include i-src, $i_a$.)

$$+v_1 + v_a - v_2 - v_3 = 0V$$

We could write an i-sum eq'n for either the node on the left or right. We only need one node. The other node is redundant.

If we use the node on the left, our eq'n is

$$-i_4 + i_q - i_1 = 0A$$

We look for components in series carrying the same current. $R_1$ and $R_2$ are in series.

$$i_4 = -i_2$$

From Ohm's law, we have

$$v_1 = i_4 R_1 = -i_2 R_1$$

$$v_2 = i_2 R_2$$

$$v_3 = i_1 R_3$$

Substituting Ohm's law and $i_4 = -i_2$ into our v-loop and i-sum eq'ns gives

$$-i_2 R_1 + v_q - i_2 R_2 - i_1 R_3 = 0V$$

$$i_2 + i_q - i_1 = 0A$$

Solving the second eq'n for $i_2$ gives

$$i_2 = i_1 - i_q$$

Substituting for $i_2$ in the first eq'n gives

$$-(i_1 - i_q)(R_1 + R_2) + V_q - i_1 R_3 = 0V$$

or $$-i_1(R_1 + R_2 + R_3) = -i_q(R_1 + R_2) - V_q$$

or $$i_1 = \frac{i_q(R_1 + R_2) + V_q}{R_1 + R_2 + R_3}$$

Consistency checks:

Set $i_q = 0A$. $i_1 = \frac{V_q}{R_1 + R_2 + R_3}$

Eq'n gives $i_1 = \frac{0 \cdot (R_1 + R_2) + V_q}{R_1 + R_2 + R_3}$ ✓

Set $V_q = 0V$. $i_1 = \frac{i_q(R_1 + R_2)}{R_1 + R_2 + R_3}$

(current divider)

Eq'n gives $i_1 = \frac{i_q(R_1 + R_2) + 0V}{R_1 + R_2 + R_3}$ ✓