Ex: Give numerical answers to each of the following questions:

a) Rationalize $\dfrac{-25j}{3-4j}$. Express your answer in rectangular form.

b) Find the rectangular form of $\left[\dfrac{(1+j)}{e^{j30^\circ}}\right]\left[\dfrac{(1+j)}{e^{-j60^\circ}}\right]^*$. (Note the asterisk that means "conjugate".)

c) Given $\omega = 2\pi$ rad/s, find the following inverse phasor:
$P^{-1} = [10(-0.866 - 0.5j)]$

d) Find the magnitude of $\dfrac{\left(4e^{j30^\circ} - \frac{1}{2}j\right)(-1-j)}{\sqrt{2}e^{j10^\circ}}$.

e) Find the real part of $\dfrac{e^{-2}}{e^{-j45^\circ}}$.

Sol'n: a) $\dfrac{-25j}{3-4j} \cdot \dfrac{3+4j}{3+4j}$ multiply top & bottom by complex conjugate of denominator

$$= \frac{(-25j)(4j) + (-25j)(3)}{3^2 + 4^2}$$

$$= \frac{100 - 75j}{25}$$

$$= 4 - 3j$$

b) $\dfrac{1+j}{e^{j30^\circ}}\left[\dfrac{1+j}{e^{-j60^\circ}}\right]^* = \dfrac{1+j}{e^{j30^\circ}} \cdot \dfrac{1-j}{e^{j60^\circ}} = \dfrac{1^2+1^2}{e^{j90^\circ}}$

But $e^{j90^\circ} = j$ and $1/j = -j$

$\therefore$ our answer is $-j2$

c) $P^{-1}[10(-0.866-0.5j)]$

$$= P^{-1}\left[10\sqrt{0.866^2+0.5^2}\, e^{j\tan^{-1}\left(\frac{-0.5}{-0.866}\right)}\right]$$

$$= P^{-1}\left[10\cdot 1\, e^{j(-150^\circ)}\right]$$

$$= P^{-1}\left[10 e^{-j150^\circ}\right]$$

$$= 10\cos(\omega t - 150^\circ)$$

$$= 10\cos(2\pi t - 150^\circ)$$

d) $$\left|\frac{\left(4e^{j30^\circ} - j\frac{1}{2}\right)(-1-j)}{\sqrt{2}\, e^{j10^\circ}}\right| = \frac{\left|4e^{j30^\circ} - j\frac{1}{2}\right|\left|-1-j\right|}{\left|\sqrt{2}\, e^{j10^\circ}\right|}$$

Magnitude of product = product of magnitudes

Now use $|e^{jx}| = 1$ for any real x, and $|a+jb| = \sqrt{a^2+b^2}$.

$$= \frac{\left|4e^{j30^\circ} - \right| \cancel{\sqrt{1^2+1^2}}}{\cancel{\sqrt{2}}} = \left|4\cos 30^\circ + j4\sin 30^\circ - \frac{j}{2}\right|$$

$$= \left|4\frac{\sqrt{3}}{2} + j4\frac{1}{2} - \frac{j}{2}\right| = \left|2\sqrt{3} + j\frac{3}{2}\right|$$

$$= \sqrt{(2\sqrt{3})^2 + \left(\frac{3}{2}\right)^2} = \sqrt{4(3) + \frac{9}{4}} = \sqrt{14\frac{1}{4}} \text{ or } \sqrt{\frac{57}{4}} = \frac{\sqrt{57}}{4}$$

e) $\text{Re}\left[\dfrac{e^{-2}}{e^{-j45^\circ}}\right] = \text{Re}\left[\dfrac{e^{-2}}{e^{-j45^\circ}} \; \dfrac{e^{j45^\circ}}{e^{j45^\circ}}\right]$

$= \text{Re}\left[e^{-2} e^{j45^\circ}\right]$ since $e^{j0^\circ} = 1$ in denominator

$= e^{-2}\,\text{Re}\left[e^{j45^\circ}\right]$ we can take positive real constants outside.

$= e^{-2}\,\text{Re}\left[\cos 45^\circ + j \sin 45^\circ\right]$

$= e^{-2}\,\text{Re}\left[\dfrac{1}{\sqrt{2}} + j\dfrac{1}{\sqrt{2}}\right]$

$= e^{-2} \cdot \dfrac{1}{\sqrt{2}}$ since $\text{Re}[a + jb] = a$