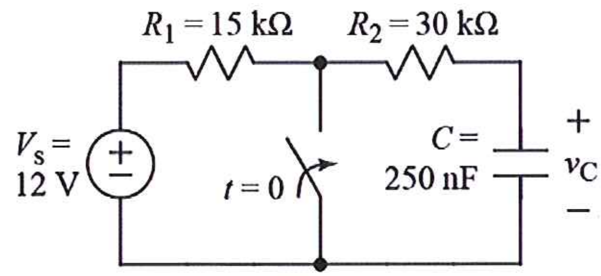
EXAM #2 prob 1 solution

Ex:



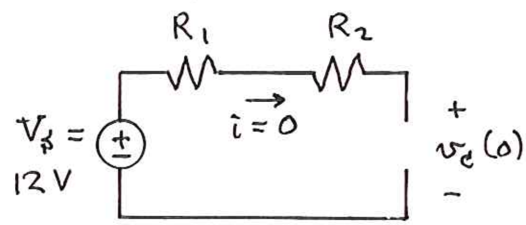
The switch is open for a long time before closing at $t = 0$.

a) Find $v_C(t = 0)$, the voltage on the capacitor at $t = 0$.

b) Find an expression for capacitor voltage, $v_C(t > 0)$. Use numerical values (rather than symbolic values) for values in the expression.

c) How long must the switch be closed until the energy stored in the capacitor is $w_C = 12.5\ \mu J$?

sol'n: a) At $t=0$, the switch has been open for a long time and the C has charged. No current is flowing into the C, so it acts like an open circuit.



No current flows thru the R's, so there is no V-drop across the R's. Thus, $v_c(0) = V_s = 12V$.

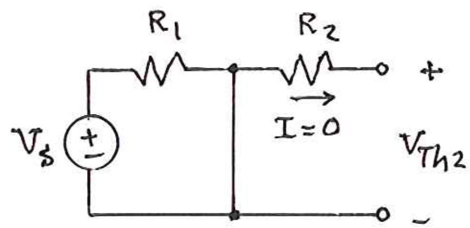
$v_c(0) = 12V$

b) We use the general formula for RC circuits:

$$v_c(t>0) = v_c(t\to\infty) + [v_c(0) - v_c(t\to\infty)]e^{-t/R_{Th2}C}$$

where R_{Th2} is the Thevenin resistance seen from the terminals where the C is connected using the switch in its position after $t=0$.

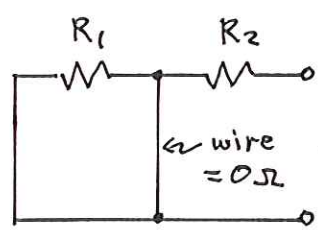
Also, $v_c(t\to\infty) = v_{Th2}$, the Thevenin voltage for the circuit after $t=0$ (from terminals where C is connected).



We have no current in R_2, so there is no v-drop across R_2. There is also no v-drop on the wire. For a v-loop on the right side, we must have $V_{Th2} = 0V$ since the wire and R_2 have 0V drops.

$$v_c(t\to\infty) = v_{Th2} = 0V$$

For R_{Th2}, we turn off V_s, which becomes a wire.



$\Leftarrow R_{Th2} = R_2$ since $0\Omega \| R_1 = 0\Omega$

$= 30k\Omega$

Our time constant is $\tau = 30k\Omega \cdot 250nF = 7.5\ ms$

$$v_c(t>0) = v_c(0)e^{-t/\tau} = 12V\, e^{-t/7.5ms}$$

c) Energy for a C is $w_c = \frac{1}{2} C v_c^2$.

$$12.5\,\mu J = \left(\frac{1}{2}\right) 250\,nF \cdot v_c^2$$

$$v_c = \sqrt{\frac{12.5\,\mu J}{\left(\frac{1}{2}\right) 250\,nF}} = \sqrt{\frac{12.5\,\mu}{125\,n}}\ V = \sqrt{100}\ V = 10\,V$$

So we find t when $v_c = 10\,V$.

$$10\,V = 12V e^{-t/7.5\,ms}$$

or

$$\frac{10}{12} = e^{-t/7.5\,ms}$$

or

$$\ln \frac{10}{12} = -t/7.5\,ms$$

or

$$t = 7.5\,ms \ln\left(\frac{12}{10}\right) = 7.5\,ms \cdot (0.182) \doteq 1.37\,ms$$

$$t = 1.37\,ms$$