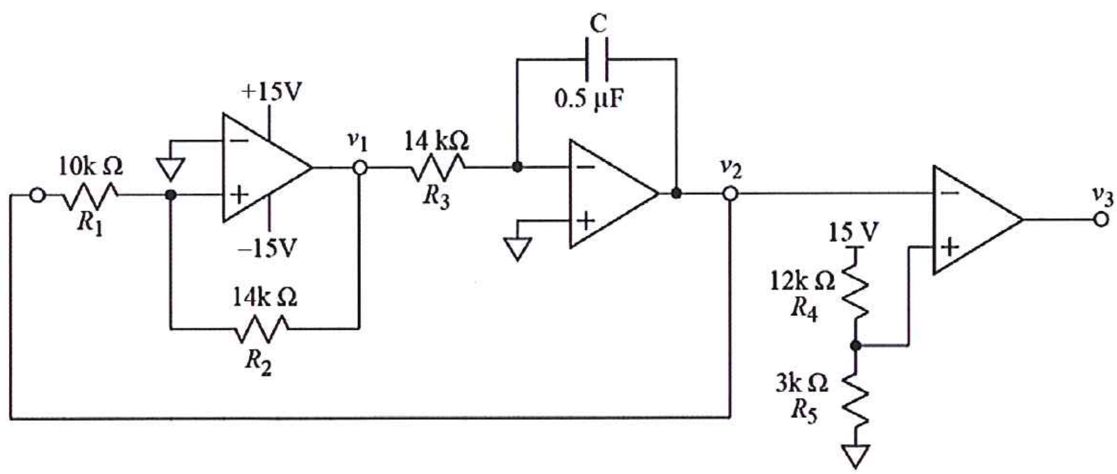
Ex:



The above circuit is from Lab 4, but a number of circuit changes have been made. Note that voltage v_2 is a triangle waveform, and v_3 is a Pulse-Width Modulation (PWM) waveform.

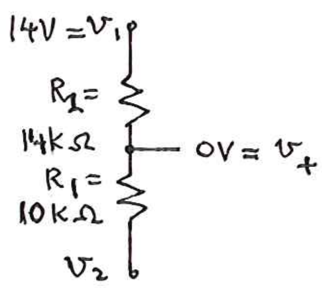
Plot v_2 and v_3. Assume $v_2 = 0V$ at $t = 0$.

a) We start by finding the value of v_2 where the first comparator op-amp switches.

Suppose v_1 is high. $v_1 = 15V$ supply $- 1V = 14V$. That is, $v_1 = 14V$, which is the positive rail voltage.

v_1 will switch when the voltages at the + and − inputs are equal. That is, v_1 switches when $v_+ = v_- = 0V$.

We have the following picture:



We have $v_+ = \dfrac{v_1 R_1}{R_1 + R_2} + \dfrac{v_2 R_2}{R_1 + R_2}$

Now we solve for v_2.

$$v_+ = \frac{v_1 R_1}{R_1 + R_2} + \frac{v_2 R_2}{R_1 + R_2}$$

or

$$0V = \frac{v_1 R_1}{R_1 + R_2} + \frac{v_2 R_2}{R_1 + R_2}$$

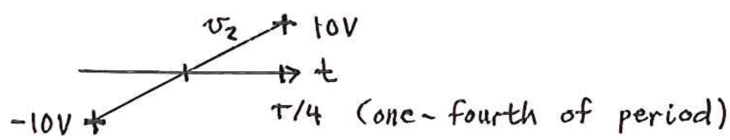
or

$$0V = v_1 R_1 + v_2 R_2$$

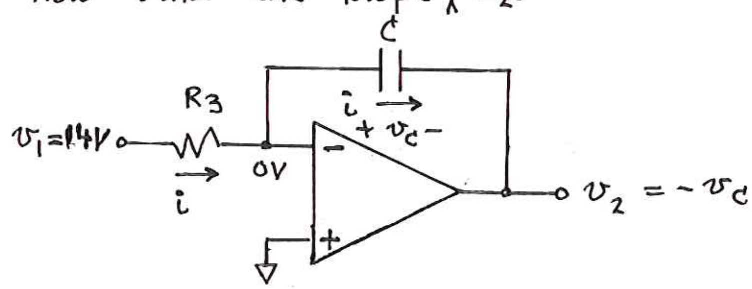
or

$$v_2 = -v_1 \frac{R_1}{R_2} = -14V \cdot \frac{10k\Omega}{14k\Omega} = -10V$$

By symmetry, v_2 will switch at $\pm 10V$.



We now find the slope of v_2.



When v_1 is high, we have current $i = \frac{v_1}{R_3}$

because $V_- = V_+ = 0V$.

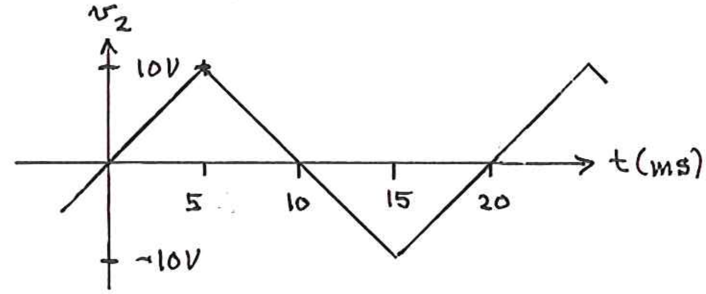
Current i charges C: $v_c(t) = \frac{1}{C}\int_0^t i(t)dt + v_c(0)$

Using $v_c(0) = 0V$, since $v_2(0) = 0$, we have

$$v_c(T/4) = 10V = \frac{1}{C}\int_0^t \frac{v_1}{R_3}dt = \frac{1}{R_3 C} v_1 t = \frac{14V\, t}{14k\Omega \cdot \frac{1}{2}\mu F}$$

or

$$v_c(T/4) = 10V = 2kV/s \cdot t \Rightarrow t = 5ms$$

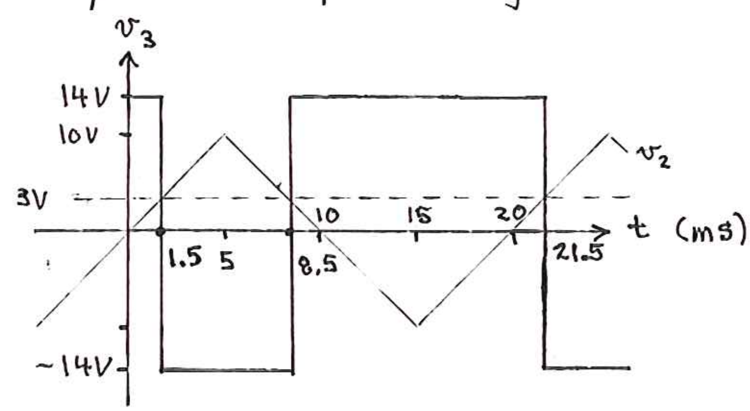


b) v_3 is the output of a comparator. v_3 will be equal to $+v_{rail} = 15V - 1V = 14V$ when $v_+ > v_-$. That is, $v_+ > v_2$.

We have a v-divider for v_+:

$$v_+ = 15V \cdot \frac{3k\Omega}{3k\Omega + 12k\Omega} = 3V$$

So the comparator output is high when $v_2 < 3V$.



The falling edge of v_3 occurs when $v_2 = 3V$.

$$v_2 = \underbrace{\frac{10V}{5ms}}_{\text{slope}} t = 3V \quad \Rightarrow \quad t = \frac{3V}{10V}(5ms) = 1.5ms$$