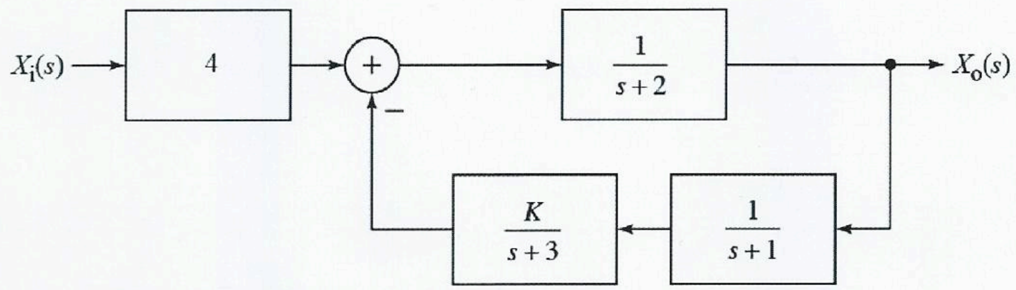
Ex:

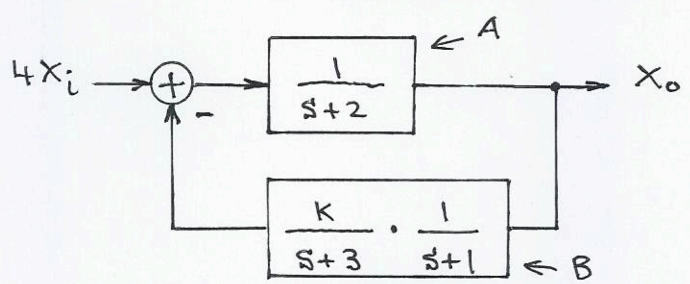


a) Find the transfer function, $H(s) = \dfrac{X_o(s)}{X_i(s)}$, for the above system.

b) For what values of K is the system on the verge of being unstable? (Hint: the system is on the verge of being unstable when one of the characteristic roots is at $s = 0$.)

sol'n: a) The box out front multiplies the transfer function of the rest of the system by 4.

For the rest of the system we may combine the boxes in the feedback into a single box.



For this configuration, the transfer function is

$$\frac{X_o(s)}{4X_i(s)} = \frac{A}{1+AB} = \frac{\frac{1}{s+2}}{1+\frac{1 \cdot K \cdot 1}{(s+2)(s+3)(s+1)}}$$

so

$$H(s) \equiv \frac{X_o(s)}{X_i(s)} = \frac{\frac{4}{s+2}}{1+\frac{K}{(s+1)(s+2)(s+3)}}$$

b) We find the characteristic roots by writing $H(s)$ as a ratio of polynomials in s and setting the denominator to zero.

$$H(s) = \frac{\frac{4}{s+2}}{1 + \frac{K}{(s+1)(s+2)(s+3)}} \cdot \frac{(s+1)(s+2)(s+3)}{(s+1)(s+2)(s+3)}$$

Note that we multiply on top and bottom by the denominator of the denominator.

$$H(s) = \frac{4(s+1)(s+3)}{(s+1)(s+2)(s+3) + K}$$

Setting the denominator equal to zero gives:

$$(s+1)(s+2)(s+3) + K = 0$$

or

$$(s+1)(s^2+5s+6) + K = 0$$

or

$$s^3 + 6s^2 + 11s + 6 + K = 0$$

To have a root at $s=0$, we must have a polynomial from which can factor out $(s-0)$ or s. This means we need to elimate the constant term, $6+K$.

$$6 + K = 0 \quad \text{or} \quad K = -6$$

Note: We may verify that the other roots are stable when $K=-6$:

$$s^3+6s^2+11s = s(s+3+j\sqrt{2})(s+3-j\sqrt{2})$$

So roots are $s=0$, $s=-3-j\sqrt{2}$, and $s=-3+j\sqrt{2}$. The real parts are less than or equal to 0. ✓