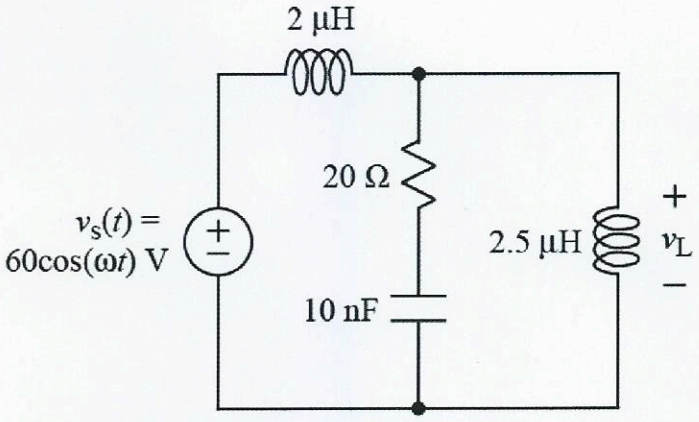
Exam #4 Prob 1 solution

Ex:



ω = 10 Mr/s

a) Draw a frequency-domain model for the above circuit.

b) Find the value of phasor voltage $\mathbf{V}_L$ across the 2.5 μH inductor.

c) Find a numerical expression for the value of $v_L(t)$.

sol'n: a) We first calculate impedance values.

$$j\omega L_1 = j\,10M\,2\mu\,\Omega = j\,20\,\Omega$$

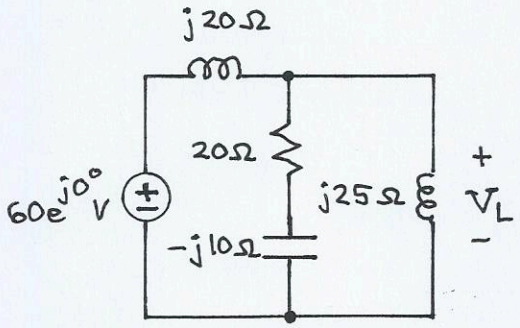
$$\frac{1}{j\omega C} = \frac{1\ \Omega}{j\,10M\,10n} = -j\,10\,\Omega$$

$$j\omega L_2 = j\,10M\,2.5\mu\,\Omega = j25\,\Omega$$

Next, we determine the phasor for the voltage source.

$$V_s = 60e^{j0^\circ}\ V$$

We draw the frequency-domain circuit using phasors and impedances.



b) We have voltage divider consisting of

$$z_1 = j20\ \Omega \quad \text{and} \quad z_2 = (20 - j10) \parallel j25\ \Omega$$

Note that V_L is also the voltage across z_2.

We now find z_2.

$$z_2 = (20 - j10) \parallel j25\ \Omega = 5\Omega \cdot (4 - j2) \parallel j5$$

or

$$z_2 = 5\Omega\ \frac{(4 - j2)\, j5}{4 - j2 + j5} = 5\Omega\ \frac{10 + j20}{4 + j3}$$

or

$$z_2 = 25\Omega\ \frac{2 + j4}{4 + j3} = 25\Omega\ \frac{2 + j4}{4 + j3} \cdot \frac{4 - j3}{4 - j3}$$

or

$$z_2 = 25\Omega\ \frac{(2 + j4)(4 - j3)}{4^2 + 3^2} = \frac{25\Omega}{25}\,(8 + 12 - j6 + j16)$$

or

$$z_2 = 20 + j10\ \Omega$$

Now for the voltage divider.

$$V_L = 60e^{j0°}\,V\ \frac{z_2}{z_1 + z_2} = 60e^{j0°}\,V\ \frac{20 + j10}{20 + j10 + j20}$$

or

$$V_L = 60e^{j0°}\,V\ \frac{20 + j10}{20 + j30}$$

Polar form is convenient for our calculation.

$$20 + j10 = 10(2+j) = 10\sqrt{2^2+1^2}\, e^{j \tan^{-1} \frac{1}{2}}$$

$$" \quad = 10\sqrt{5}\, e^{j\,26.6^\circ}$$

$$20 + j30 = 10(2+j3) = 10\sqrt{2^2+3^2}\, e^{j \tan^{-1} \frac{3}{2}}$$

$$" \quad = 10\sqrt{13}\, e^{j\,56.3^\circ}$$

Using these results, we complete the calculation of V_L.

$$V_L = 60e^{j0^\circ}\ V\ \frac{10\sqrt{5}\, e^{j\,26.6^\circ}}{10\sqrt{13}\, e^{j\,56.3^\circ}}$$

$$V_L = \frac{60\sqrt{5}}{\sqrt{13}}\, e^{j(26.6^\circ - 56.3^\circ)} \qquad V = 37.2\, e^{-j\,29.7^\circ}\ V$$

$$V_L = 37.2\, e^{-j\,29.7^\circ}\ V$$

c) Converting back to the time domain involves re-introducing the $\cos(\omega t)$ term.

$$v_L(t) = 37.2 \cos(10\,Mr/s \cdot t - 29.7^\circ)\ V$$