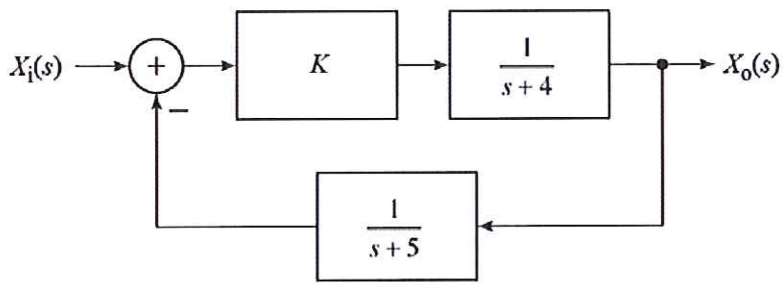
Ex:



a) Find the transfer function, $H(s) = \dfrac{X_o(s)}{X_i(s)}$, for the above system.

b) For what values of K is the system stable? (Consider positive and negative values of K.)

sol'n: a) The forward path is $K\left(\frac{1}{s+4}\right) = A(s)$, and the feedback path is $\frac{1}{s+5} = B(s)$. Using the formula for the transfer function of a standard feedback system yields the following expression:

$$H(s) = \frac{A(s)}{1 + A(s)B(s)} = \frac{K/(s+4)}{1 + \frac{K}{s+4}\frac{1}{s+5}}$$

b) The first step in determining stability is to write $H(s)$ as a ratio of polynomials in s.

$$H(s) = \frac{\frac{K}{s+4}}{1 + \frac{K}{s+4} \cdot \frac{1}{s+5}} \frac{(s+4)(s+5)}{(s+4)(s+5)} = \frac{K(s+5)}{(s+4)(s+5) + K}$$

The system is stable when the roots of the denominator have non-positive real parts.

$$(s+4)(s+5) + K = 0$$

or

$$s^2 + 9s + 20 + K = 0$$

The quadratic equation gives the values of the roots.

$$s = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - (20+K)}$$

For the real part of s to be positive, the square root would have to be greater than $+9/2 = +4.5$. The extremum value of K before the system becomes unstable occurs when the square roots equals 4.5.

$$\sqrt{\left(\frac{9}{2}\right)^2 - (20+K)} = 4.5$$

or

$$\cancel{\left(\frac{9}{2}\right)^2} - (20+K) = \cancel{4.5^2}$$

or $\quad -(20+K) = 0$

or $\quad K = -20$

The system is stable for $\boxed{K \geq -20}$