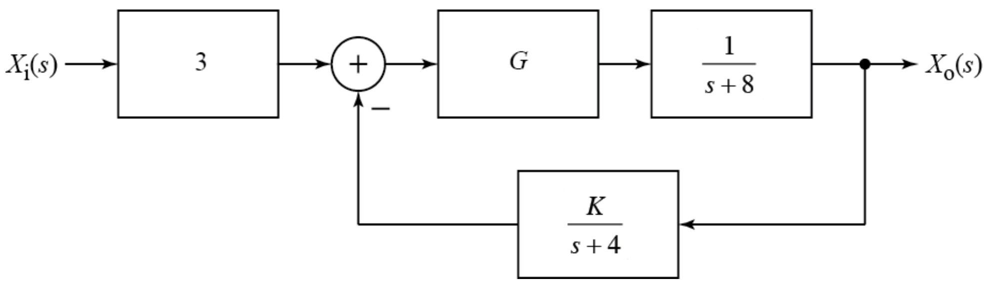
Exam #3 Prob 1 solution

Ex:



a) Find the transfer function, $H(s) = \dfrac{X_o(s)}{X_i(s)}$, for the above system.

b) If G = 100,000, for what values of K is the system stable? (Consider positive and negative values of K.)

SOL'N: a) We multiply the input to a box by the quantity in the box to get the output of the box. We can combine boxes in series by multiplying the quantities they contain. This reduces the above system to one with a gain of 3 followed by a system with a single forward-path box containing $G\dfrac{1}{s+8}$.

For a forward path of $G\dfrac{1}{s+8}$ and feedback of K, we employ the formula that the transfer function is given by the forward path divided by one plus the product of the forward path and feedback path. Combining this with the gain of 3 in the first box, we obtain $H(s)$:

$$H(s) = \frac{X_o(s)}{X_i(s)} = \frac{3G\dfrac{1}{s+8}}{1 + G\dfrac{1}{s+8}K\dfrac{1}{s+4}}$$

We simplify the expression by multiplying top and bottom by the denominator of the denominator, namely, $(s + 8)(s + 4)$.

$$H(s) = \frac{3G\dfrac{1}{s+8}}{1 + G\dfrac{1}{(s+8)(s+4)}K} \cdot \frac{(s+8)(s+4)}{(s+8)(s+4)} = \frac{3G(s+4)}{(s+8)(s+4)+GK}$$

Next, we write $H(s)$ in standard form as a constant times a ratio of polynomials in s with the coefficient of the highest power of s being unity.

$$H(s) = \frac{3G(s+4)}{s^2 + 12s + 32 + GK} = 3G \cdot \frac{s+4}{s^2 + 12s + 32 + GK}$$

b) The system is stable when the real parts of the roots of the denominator are all negative. (This corresponds to time-domain solutions of the form e^{st} that will decay over time. If the real part of s is positive, we get a growing exponential.)

To find the roots, we set the polynomial in the denominator equal to zero. Note that the constant term $3G$ does not change the value of the roots. Here, we have two roots since our denominator is second-order.

$$s^2 + 12s + 32 + GK = 0$$

We use the quadratic equation solution:

$$s_{1,2} = -\frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 - (32 + GK)} = -6 \pm \sqrt{6^2 - (32 + 100{,}000 \cdot K)}$$

The value of the square root determines whether the root for the second root (meaning the "plus-the-square-root root") will have a positive real part. If $K > 0$, then the magnitude of the square root is less than 6 and the second root will have a negative real part.

If the value of K is negative and $32 + 100{,}000K < 0$, the magnitude of the square root will be greater than 6, and the second root will be a positive number, meaning it has a positive real part.

So the system is stable precisely when $32 + 100{,}000K > 0$ or

$$K > -\frac{32}{100{,}000} = -320\,\mu$$