EX: Show the minimum logic circuit (i.e., using logic gates) to implement the following expression. You may use AND, OR, EX-OR, and NOT (inverter) gates. The optimal design has the minimum total number of gate inputs.

$$F = AB + \bar{A}C + \bar{C}$$

SOL'N: We have the following truth table for F versus A, B, and C:

A	B	C	AB	$\bar{A}C$	$\bar{C}$	F
0	0	0	0	0	1	1
0	0	1	0	1	0	1
0	1	0	0	0	1	1
0	1	1	0	1	0	1
1	0	0	0	0	1	1
1	0	1	0	0	0	0
1	1	0	1	0	1	1
1	1	1	1	0	0	1

One simple solution is to use a 3-input NAND gate:

$$F = \overline{A\bar{B}C}$$

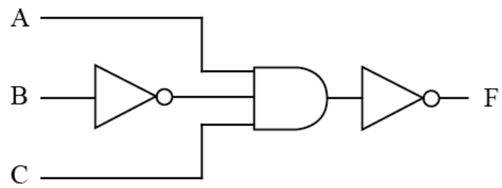
Applying De Morgan's theorem, we can use a 3-input OR gate:

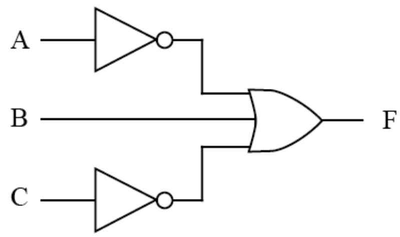
$$F = \bar{A} + B + \bar{C}$$

Applying De Morgan's theorem to A and C, we get another solution with the same number of gate inputs:

$$F = \overline{AC} + B$$

The three designs, shown below, all have five gate inputs:


A
B
C
F


A
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C
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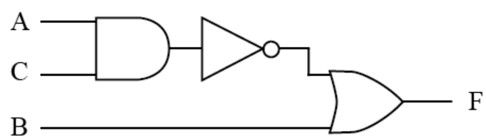
A
C
B
F