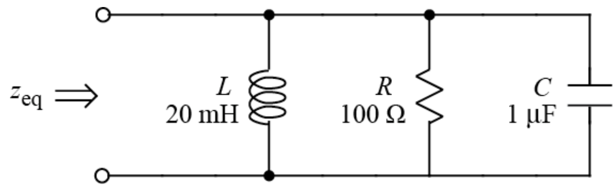
Ex:



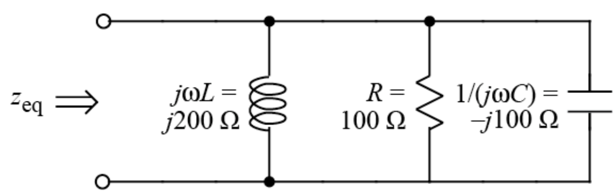
Find the numerical value of the equivalent impedance, z_{eq}, for the above circuit. Frequency $\omega = 10$ kr/s. Express your answer in polar form.

Sol'n: We convert to the frequency domain by computing impedance values.

$$j\omega L = j(10\,\text{k})(20\,\text{m})\Omega = j200\,\Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j(10\text{k})(1\mu)}\Omega = -\frac{j}{10\,\text{m}} = -j100\,\Omega$$

Our frequency domain circuit:



We treat impedance just as we would resistance. We have R in parallel with C and L.

$$z_{eq} = 100\,\Omega \,\|\, -j100\,\Omega \,\|\, j200\,\Omega = 100\,\Omega \cdot (1 \,\|\, -j \,\|\, j2)$$

We combine the two imaginary impedances first.

$$-j \,\|\, j2 = \frac{1}{\dfrac{1}{-j} + \dfrac{1}{j2}} \cdot \frac{j}{j} = \frac{j}{-1 + \dfrac{1}{2}} = -j2$$

So we have the following impedance:

$$z_{eq} = 100\,\Omega \cdot (1 \,\|\, -j2) = 100\Omega \cdot \frac{1}{\dfrac{1}{1} + \dfrac{1}{-j2}} = 100\Omega \cdot \frac{1}{1 + \dfrac{j}{2}}$$

This expression is simple enough for us to use rationalization:

$$z_{\text{eq}} = 100\Omega \cdot \frac{1}{1+\frac{j}{2}} \cdot \frac{1-\frac{j}{2}}{1-\frac{j}{2}} = 100\Omega \cdot \frac{1-\frac{j}{2}}{1^2 + \left(\frac{1}{2}\right)^2} = 100\Omega \cdot \frac{1-\frac{j}{2}}{\frac{5}{4}}$$

or

$$z_{\text{eq}} = 100\Omega \cdot \frac{4}{5} \cdot \left(1-\frac{j}{2}\right) = 80\Omega \cdot \left(1-\frac{j}{2}\right) = 80\Omega - j40\Omega$$

In polar form:

$$z_{\text{eq}} = 80\Omega \cdot \left(1-\frac{j}{2}\right) = 80\Omega \cdot \sqrt{1^2 + \left(\frac{1}{2}\right)^2}\, e^{j\tan^{-1}\left(\frac{-1/2}{1}\right)}$$

or

$$z_{\text{eq}} = 80\Omega \cdot \frac{\sqrt{5}}{2} e^{-j26.6^\circ} \doteq 40\sqrt{5} e^{-j26.6^\circ}\,\Omega = 89.4 e^{-j26.6^\circ}\,\Omega$$