ECE 1250 homework # P1

a

Read about complex numbers and phasors in your textbook (sections 2.26 & 2.27, starting on p.159).

1. For the complex numbers $z_1 := -4 + 5\cdot j$ and $z_2 := 2 + 4\cdot j$ Determine the following

   a) Does $|z_1 \cdot z_2|$ equal $|z_1| \cdot |z_2|$?

   b) Does $\left|\frac{z_1}{z_2}\right|$ equal $\frac{|z_1|}{|z_2|}$?

   c) Does $|z_1 + z_2|$ equal $|z_1| + |z_2|$?

2. a) Find the phasor for $v(t) = 8.4\cdot\cos(100\cdot t - 90\cdot\text{deg})$ Express in both forms, polar and rectangular.

   b) The phasor representation of a current is $I := (5 + j\cdot 12)\cdot\mu A$ Find the time-domain representation, i(t). $f := 600\cdot Hz$

3. Add or subtract the sinusoidal voltages using phasors. Draw a phasor diagram which shows all 3 phasors, and give your final answer in time domain form.

   a) $v_1(t) = 1.5\cdot V\cdot\cos(\omega\cdot t + 10\cdot\text{deg})$ $v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 25\cdot\text{deg})$ Find $v_3(t) = v_1(t) + v_2(t)$

   b) $v_1(t) = 1.5\cdot V\cdot\cos(\omega\cdot t + 10\cdot\text{deg})$ $v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 25\cdot\text{deg})$ Find $v_4(t) = v_1(t) - v_2(t)$

   you may add V_4 to the phasor diagram you've already drawn for part a).

4. Express the impedance of a $5.2mH$ inductor at $60\ Hz$ in polar form.

5. a) A capacitor impedance has a magnitude of 240Ω at a frequency of $1.8kHz$. What is the value of capacitor?

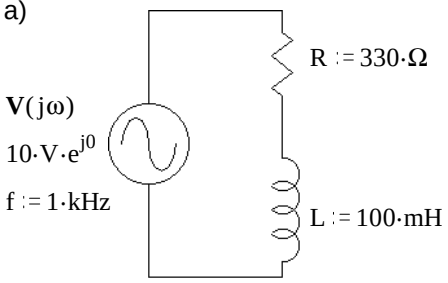
   b) What value inductor has the same impedance magnitude at the same frequency?

   c) Find the reactance (magnitude of the impedance with + or - sign) of this capacitor and this inductor at $3.6kHz$?

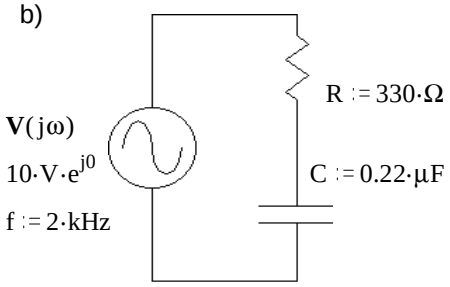
   d) What would be the total impedance of this inductance and this capacitance connected in series at $2.7kHz$?

6. Find Z_{eq} in each case.

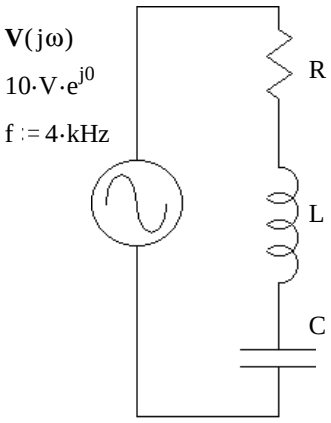
a)



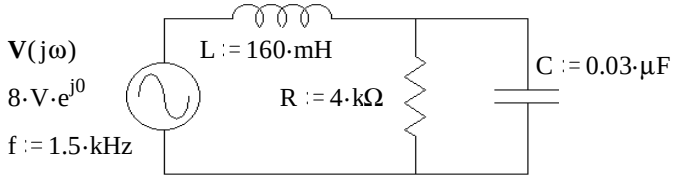
b)



c)



d)

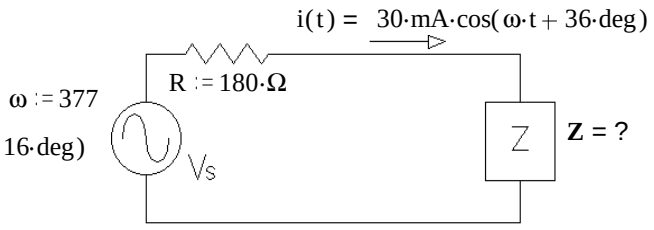


7. Find the current $\mathbf{I}(j\omega)$ in each case above.

8. a) Find $\mathbf{Z}$. Hint: Find the total impedance ($R+\mathbf{Z}$) first.

   b) Which leads, current or voltage?

   c) By how much?
   I.E. what is the phase angle between the voltage and current?



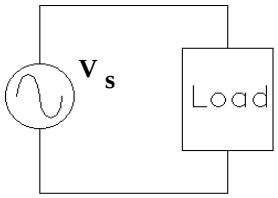
$v_s(t) = 14\cdot V\cdot\cos(\omega\cdot t + 16\cdot\text{deg})$

9. a) A resistor and a capacitor are connected in <u>series</u> to create an impedance of $\mathbf{Z} := 50\cdot\Omega\cdot e^{-j\cdot 66\cdot deg} = 50\Omega\ \underline{/-66^o}$ at a frequency $f := 3\cdot kHz$ Find R and C.

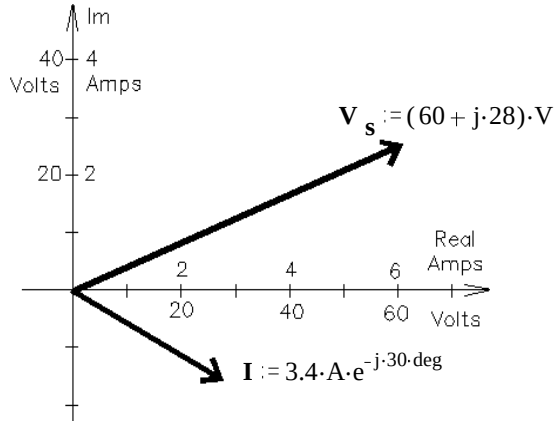
b) A resistor and a capacitor are connected in <u>parallel</u> to create an impedance of $\mathbf{Z} := 50\cdot\Omega\cdot e^{-j\cdot 66\cdot deg} = 50\Omega\ \underline{/-66^o}$ at a frequency $f := 3\cdot kHz$ Find R and C.

Hint: invert $\mathbf{Z_{eq}}$, Instead of solving this: $50\ \underline{/-66deg} = \dfrac{1}{\frac{1}{R} + j\cdot\omega\cdot C}$ solve this: $\dfrac{1}{Z_{eq}} = 0.02\underline{/66deg} = \dfrac{1}{R} + j\cdot\omega\cdot C$

10. The phasor diagram at right shows the voltage and current in the circuit below



Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz.

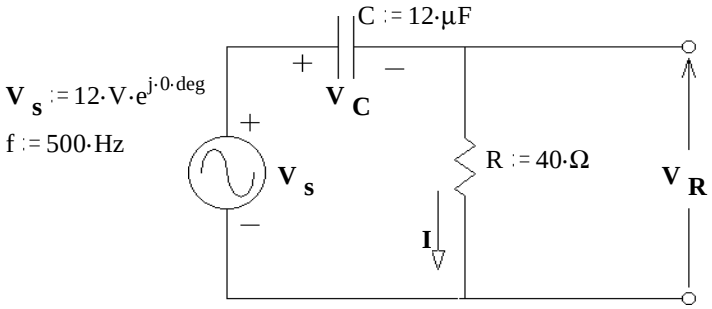


a) What is the magnitude of the impedance?

b) What is the value of the resistor?

c) What is the reactive component (type and value)?

11. For the circuit shown, draw a phasor diagram showing $\mathbf{V_S}$, $\mathbf{I}$, $\mathbf{V_R}$, and $\mathbf{V_C}$. Draw the voltages to scale so that you can show that they obey KVL.



<u>Answers</u>

1. a) 28.636 = 28.636 Yes b) 1.432 = 1.432 Yes
   c) 9.22 not = 10.875 No
2. a) $8.4\ \underline{/-90^o} = 8.4\cdot e^{-j\cdot 90\cdot deg} = -8.4j$ b) $13\cdot\mu A\cdot\cos(3770\cdot t + 67.4\cdot deg)$
3. a) $v_1(t) + v_2(t) = 4.67\cdot\cos(\omega\cdot t + 20.2\cdot deg)\cdot V$
   b) $v_1(t) - v_2(t) = 1.794\cdot\cos(\omega\cdot t - 142.5\cdot deg)\cdot V$
4. $1.96\ \Omega\ \underline{/90^o}$
5. a) $0.368\cdot\mu F$ b) $21.2\cdot mH$ c) $-120\cdot\Omega$ $480\cdot\Omega$ d) $200\cdot j\cdot\Omega$
6. a) $(330 + 628.3\cdot j)\cdot\Omega = 709.7\Omega\ \underline{/62.29^o}$
   b) $(330 - 361.7\cdot j)\cdot\Omega = 489.6\Omega\ \underline{/-47.63^o}$
7. a) $(6.6 - 12.5\cdot j)\cdot mA = 14.1mA\ \underline{/-62.29^o}$
   b) $(13.8 + 15.1\cdot j)\cdot mA = 20.4mA\ \underline{/47.63^o}$
   d) $4.4mA\ \underline{/15.2^o}$
8. a) $259 - 160\cdot j$ b) The current leads the voltage c) 20^o
9. a) $20.34\cdot\Omega$ $1.16\cdot\mu F$ b) $123\cdot\Omega$ $0.969\cdot\mu F$
10. a) $19.5\cdot\Omega$ b) $11.2\cdot\Omega$ c) inductor $42.3\cdot mH$

c) $R + \left(\omega\cdot L - \dfrac{1}{\omega\cdot C}\right)\cdot j$ d) $1.82k\Omega\ \underline{/-15.2^o}$

c) $\dfrac{10\cdot V}{\sqrt{R^2 + \left(\omega\cdot L - \dfrac{1}{\omega\cdot C}\right)^2}}$ angle: $-\operatorname{atan}\left(\dfrac{\omega\cdot L - \dfrac{1}{\omega\cdot C}}{R}\right)$

11.