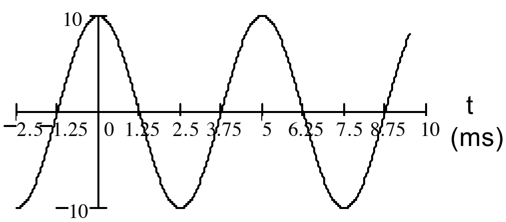
A. Stolp
1/26/12

Sine waves

v(t) A sine wave:



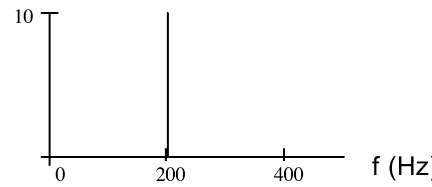
$amplitude := 10 \cdot V$

$T := 5 \cdot ms$

$f := \frac{1}{T}$ $\quad f = 200 \cdot Hz$

$\omega := 2 \cdot \pi \cdot f$ $\quad \omega = 1257 \cdot \frac{rad}{sec}$

The "frequency" domain:

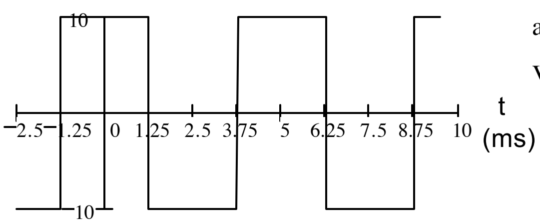


No bandwidth = No Information

Periodic waves

Fourier series: Any periodic waveform can be represented by a series of sinewaves of different frequencies.

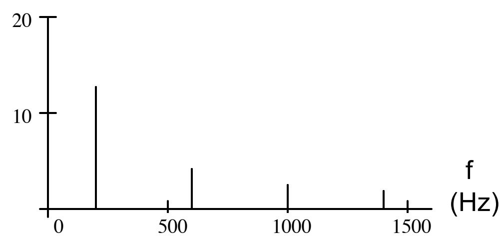
v(t) A square wave:



$amplitude := 10 \cdot V$

$V_{RMS} := 10 \cdot V$

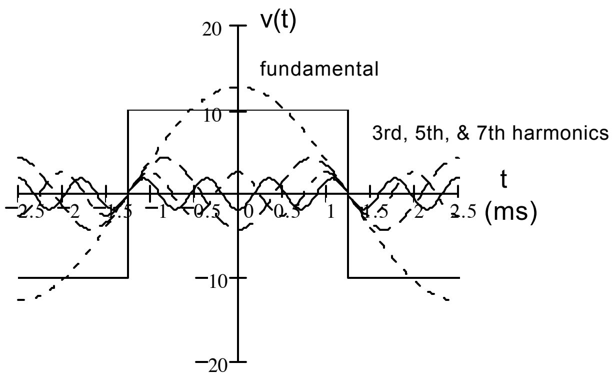
The "frequency" domain:

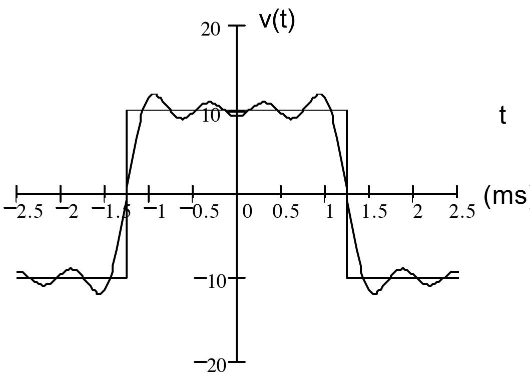


Still no bandwith

$$v(t) = \frac{4}{\pi} \cdot \cos(\omega \cdot t) - \frac{4}{3 \cdot \pi} \cdot \cos(3 \cdot \omega \cdot t) + \frac{4}{5 \cdot \pi} \cdot \cos(5 \cdot \omega \cdot t) - \frac{4}{7 \cdot \pi} \cdot \cos(7 \cdot \omega \cdot t) \quad + \, . \, - \, . \, + \, .$$

Notice that the frequency spectrum shows the amplitudes of the harmonics, but not the phases.





fundamental + 3rd, 5th, & 7th harmonics

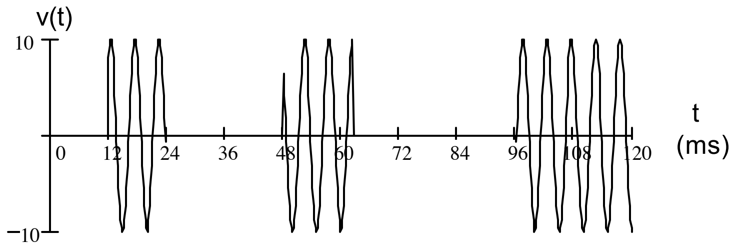
You need bandwith to transmit information

Sine waves are "pretend" signals

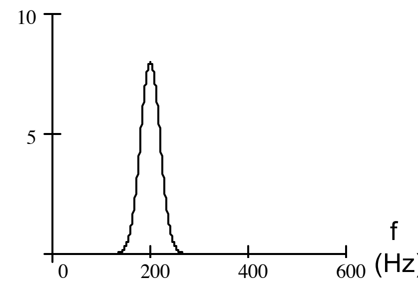
Although sine waves are not really signals, we use them to simulate signals all the time, both in calculations and in the lab. This makes sense because all signals can be thought of as being made up of a spectrum of sine waves.

However, if we change the waveform in any non-periodic way, then the spectrum will no longer be just lines, and we'll have bandwidth.

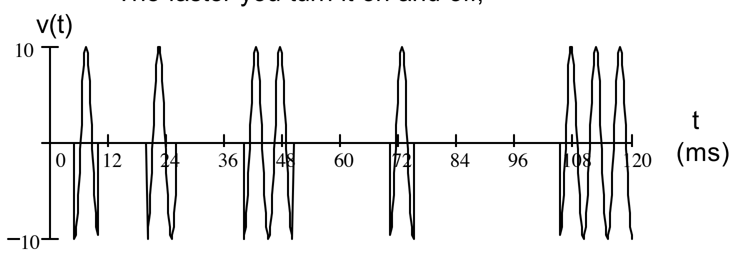
Just turning the sine wave on and off in some unpredictable way,



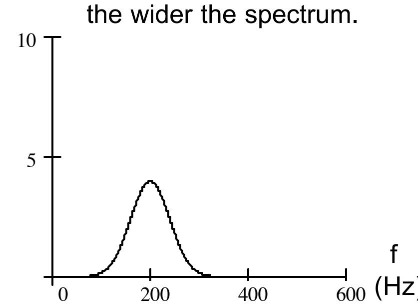
Makes the specturm widen



The faster you turn it on and off,



the wider the spectrum.



The faster things happen, the wider the bandwidth. The sharper the edges, the higher the frequencies. Obviously these two phenomina are related.

To get the spectrum of a "random" waveform you must take the Fourier Transform instead of the Fourier series.

Signal Processing

Amplification, creating a duplicate of a signal which has more power than the original.

Filtering low-pass, high-pass, band-pass, band-reject, notch, etc...

Modulation, demodulation AM, FM, phase

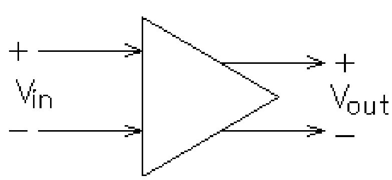
Multiplexing, frequency, time

Analog to Digital Conversion (ADC), Digital to Analog Conversion (DAC)

Etc...

Amplification

General symbol:

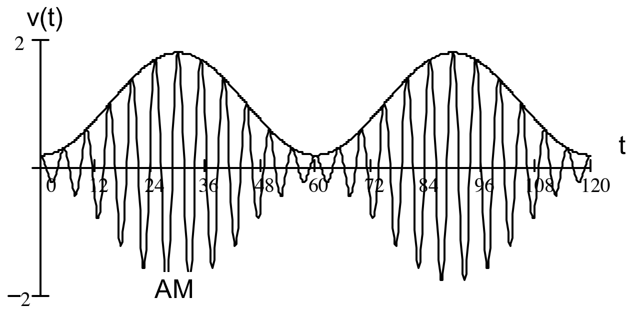


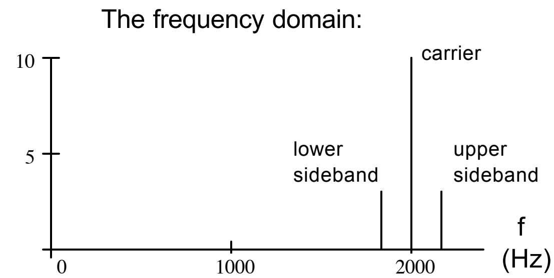
voltage "gain" = $\frac{v_{out}}{v_{in}}$

We can talk about voltage gain, current gain, and power gain.
All amplifiers must have the potential for power gain (will depend on the "load") -- Transformers are NOT amplifiers
Of course this means that all amplifiers must be connected to a power supply!

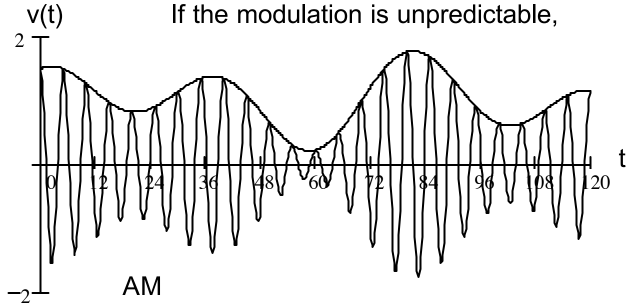
Amplifiers don't make a signal bigger, they actually make a bigger copy of the original.
If the copy is exact, then there is no "distortion". All real amplifiers have some distortion.

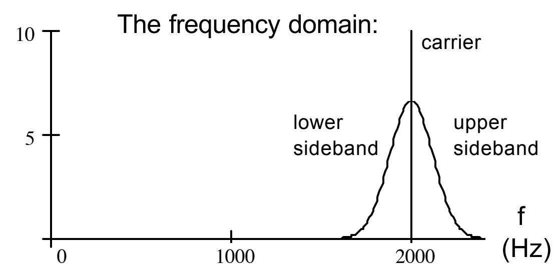
Modulation, demodulation

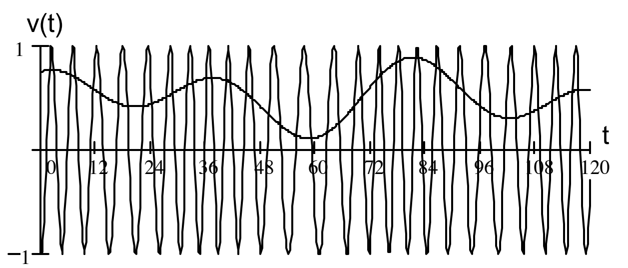




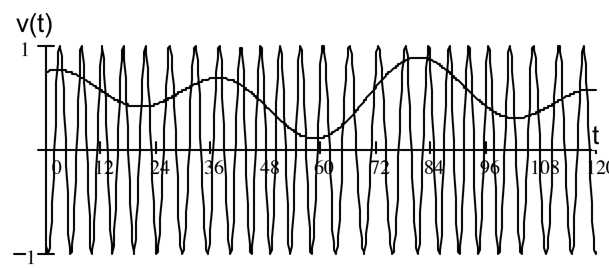
If the modulation is unpredictable, then the spectrum blurs out







FM



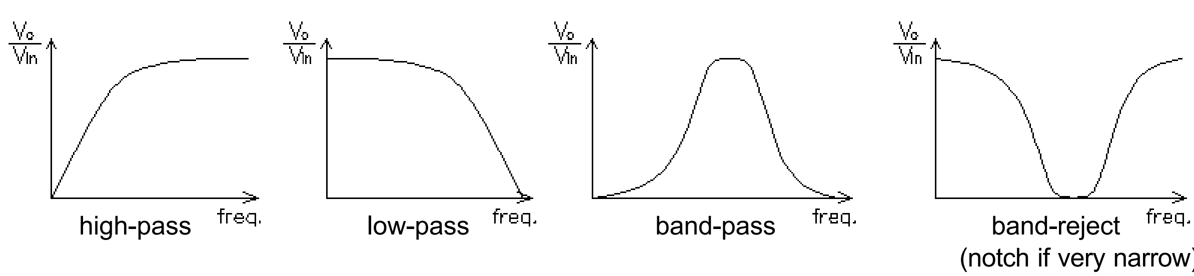
phase

Multiplexing

Frequency multiplexing (Like radio stations which each use a different carrier frequency)

Time multiplexing (Used with digital signals, the bits of one signal are sent for a short time, then the bits of another, then another, and so forth.)

Filtering

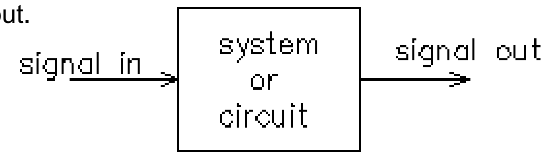


etc..

Frequency response

The "response" of a system or circuit is the output for a given input.

A "transfer function" is a mathmatical description of how the output is related to the input.



"Block diagram"

output = Transfer function x input

or... Transfer function = $\dfrac{\text{output}}{\text{input}}$

No real system or circuit treats all frequencies the same, so all real transfer functions are functions of frequency.

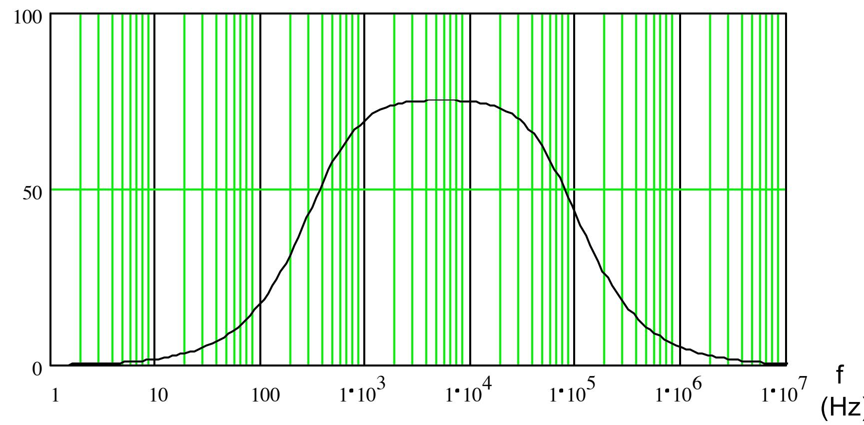
Transfer function = $H(\omega)$ or $H(f)$ or, Transfer function = $H(s)$

The transfer function can be used to describe the "frequency response" of a circuit. That is, how does the circuit respond to inputs of different frequencies.

A typical frequency response curve for a circuit we might work with:

Magnitude plot

$|H(f)|$

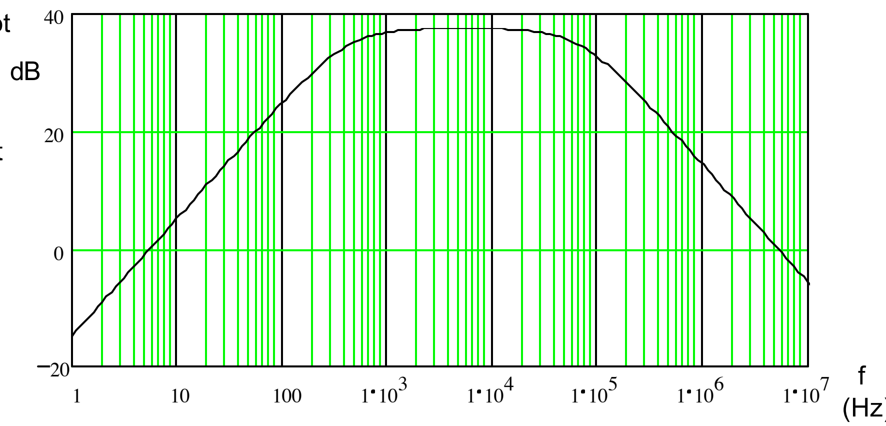


More commonly, the magnitude plot will be expressed in term of "decibels" (dB), a log function.

Magnitude plot

$|H(f)|$

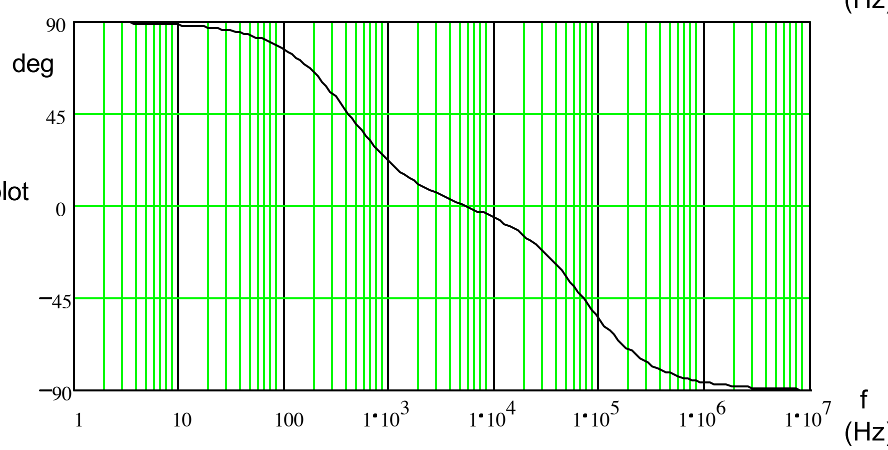
dB will be explained later



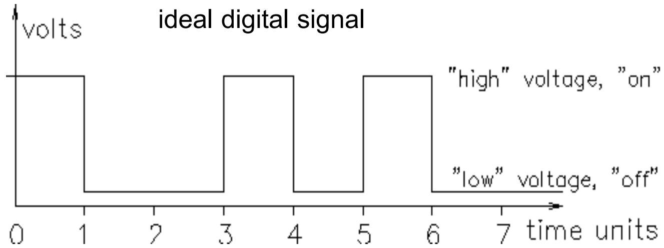
Phase angle is also part of the frequency response

Phase angle plot

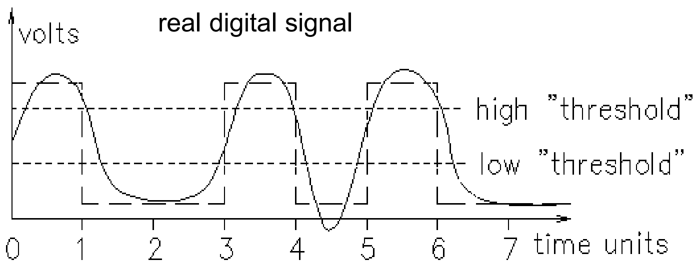
$\angle H(f)$



Digital Signals



But now we know that sharp edges = high frequencies and no system has a perfect frequency response.



digital circuits use thresholds to tell a "high" from a "low". Some levels may not be defined either way.

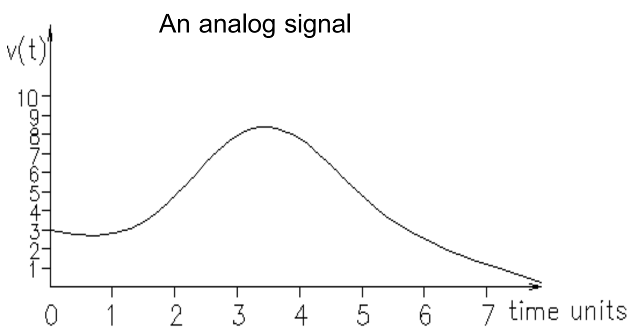
Behind all digital signals there are really analog signals.

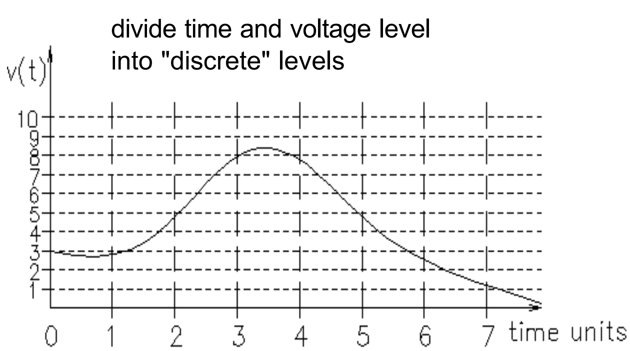
Analog - to - digital conversion

(ADC or A/D converter or A to D converter)

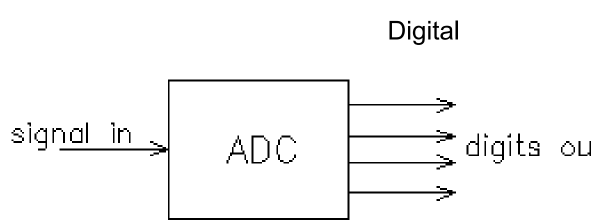
Neglecting the underlying analog nature of digital signals for the moment...

An analog signal can be represented by numbers.





time	level	binary representation
0	3	0011
1	3	0011
2	5	0101
often you can't fit to an exact level, this is called "quantization error"		
3	8	1000
4	8	1000
5	5	0101
6	3	0011
7	1	0001



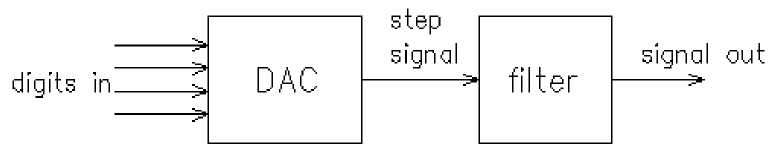
usually contains a filter and a sample & hold as well as an ADC

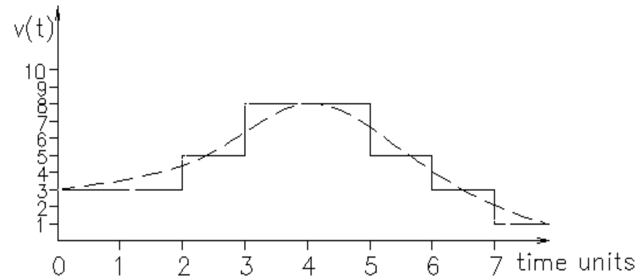
Digital - to - analog conversion

(DAC or D/A converter or D to A converter)

Take those digits and turn them back into voltage levels.

Filter the result to get a close representation to what you started with.





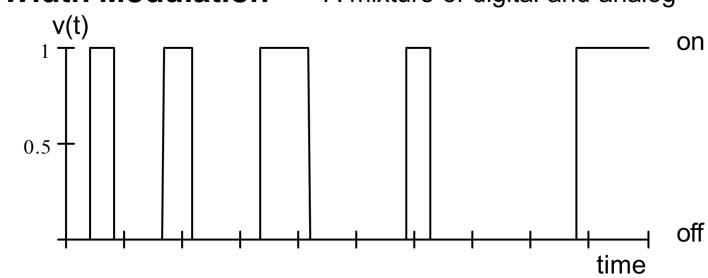
In fact, if the sampling rate is at least twice the highest frequency found in the input signal and the filter is a perfect low-pass filter, then the output can be exactly the same as the input ($\pm$ the quantization error). This is the "Nyquist" theorem.

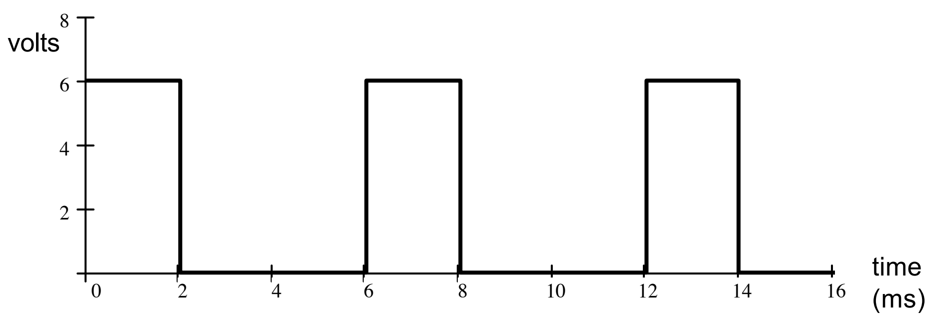
More "bits" = less quantization error, less "noise"

Faster sampling rate = higher frequency response

Pulse Width Modulation

A mixture of digital and analog

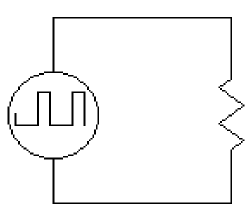




Average DC (V_{DC}) value = $\frac{6\cdot V\cdot(2\cdot ms)+0\cdot V\cdot(4\cdot ms)}{6\cdot ms} = 2\cdot V$

Duty cycle = $\frac{2\cdot ms}{6\cdot ms}\cdot 100\cdot\% = 33.33\cdot\%$

If this voltage is hooked to a resistor, as shown



$R_L := 50\cdot\Omega$

When on: $P = \frac{(6\cdot V)^2}{50\cdot\Omega} = 0.72\cdot W$

Average power: $\frac{0.72\cdot W\cdot(2\cdot ms)+0\cdot W\cdot(4\cdot ms)}{6\cdot ms} = 0.24\cdot W$

How much energy is transferred to the resistor during 6 seconds?

$W_L := 0.24\cdot W\cdot 6\cdot sec$ $\qquad W_L = 1.44\cdot joule$