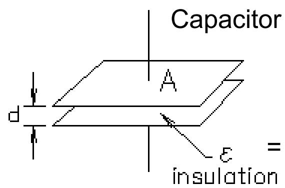
ECE 1250 Capacitor Lecture Notes

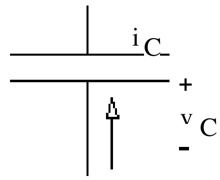
Now that we have voltages and currents which can be functions of time, it's time to introduce the capacitor.

Electrical equivalent:



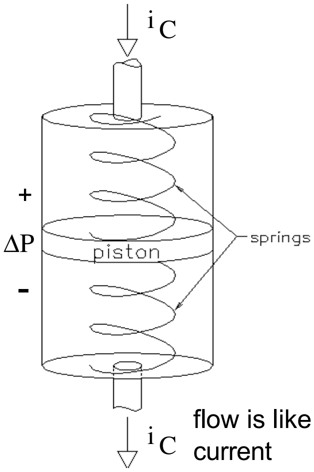
ε = permittivity

$$C = \varepsilon \cdot \frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv}$$



v_C like pressure

Fluid Model:



flow is like current

Units: $\text{farad} = \dfrac{\text{coul}}{\text{volt}} = \dfrac{\text{amp} \cdot \text{sec}}{\text{volt}}$ $\qquad \mu F = 1 \cdot 10^{-6} \cdot \text{farad} \qquad pF = 1 \cdot 10^{-12} \cdot \text{farad}$

For drawings of capacitors and info about tolerances, see Ch.3 of textbook.

Basic equations you should know: $\quad C = \dfrac{Q}{V}$

$$v_C = \frac{1}{C} \cdot \int_{-\infty}^{t} i_C \, dt = \frac{1}{C} \text{ (the accumulation of current)}$$

$$i_C = C \cdot \frac{d}{dt} v_C$$

initial voltage

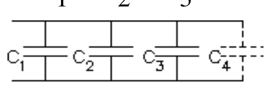
Or... $\quad v_C = \dfrac{1}{C} \cdot \displaystyle\int_0^t i_C \, dt + v_C(0)$

Or... $\quad \Delta v_C = \dfrac{1}{C} \cdot \displaystyle\int_{t_1}^{t_2} i_C \, dt$

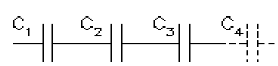
Energy stored in electric field: $\quad W_C = \dfrac{1}{2} \cdot C \cdot V_C^{\,2}$

Capacitor voltage **cannot** change instantaneously

parallel: $\quad C_{eq} = C_1 + C_2 + C_3 \quad + \ldots$



series: $\quad C_{eq} = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} \quad + \ldots}$



Capacitors are the only "backwards" components.

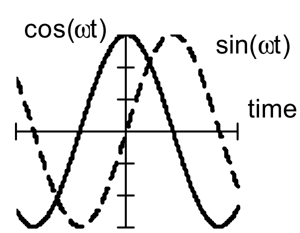
Sinusoids

$$i_C(t) = I_p \cdot \cos(\omega \cdot t)$$

$$v_C(t) = \frac{1}{C} \int i_C \, dt = \underbrace{\frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p}_{V_p} \cdot \sin(\omega \cdot t) = \underbrace{\frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p}_{V_p} \cdot \cos(\omega \cdot t - 90 \cdot \deg)$$

indefinite integral

Voltage "lags" current, makes sense, current has to flow in first to charge capacitor.

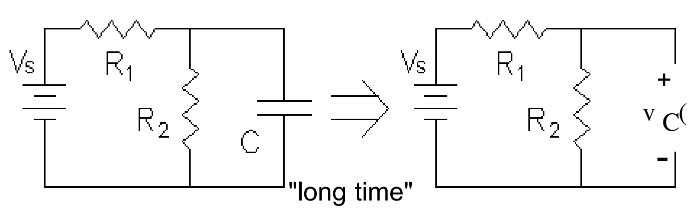


Steady-state or Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$$\frac{d}{dt} v_C = 0 \qquad i_C = C \cdot \frac{d}{dt} v_C = 0$$

no current means it looks like an open



$$v_C(\infty) = V_S \cdot \frac{R_2}{R_1 + R_2}$$

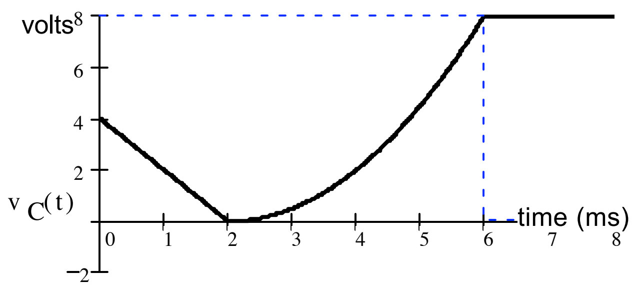
Example

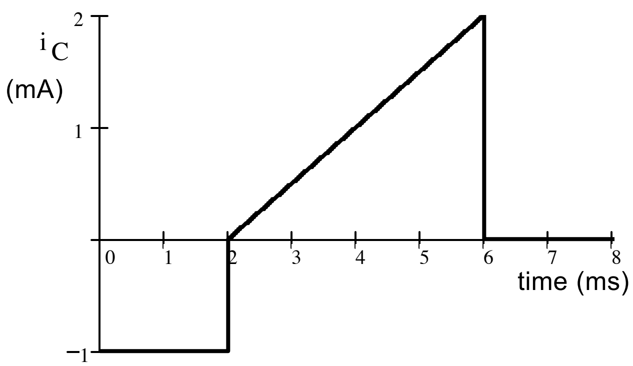
The voltage across a $0.5\ \mu F$ capacitor is shown below. Make an accurate drawing of the capacitor current. Label the y-axis of your graph (I've already done the time-axis).

The accuracy of your plot at $0, 2, 6,$ and 8 ms is important, so calculate those values and plot or label them carefully. Between those points your plot must simply be the correct shape.

$C := 0.5 \cdot \mu F$

The curve is 2nd order





1 - 2ms: $i_C = C \cdot \frac{\Delta V}{\Delta t} = 0.5 \cdot \mu F \cdot \frac{-4 \cdot V}{2 \cdot ms} = -1 \cdot mA$

2ms - 6ms: Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

$$\Delta v_C(t) = \frac{1}{C} \cdot \int_0^t i_C(t)\, dt$$

$$8 \cdot V = \frac{1}{C} \cdot \left(\frac{4 \cdot ms \cdot height}{2} \right)$$

$$height = 8 \cdot V \cdot \frac{C \cdot 2}{4 \cdot ms} = 2 \cdot mA$$

6ms - 8ms: Slope is zero, so the current must be zero.

RC first-order transient circuits

For all first order transients: $v_X(t) = v_X(\infty) + \left(v_X(0) - v_X(\infty)\right) \cdot e^{-\frac{t}{\tau}}$ $\quad i_X(t) = i_X(\infty) + \left(i_X(0) - i_X(\infty)\right) \cdot e^{-\frac{t}{\tau}}$

Find the initial condition $v_X(0)$ or $i_X(0)$

Find the capacitor voltage just before time $t = 0$, $v_C(0-)$. This will be the same just after time $t = 0$, $v_C(0+)$. The capacitor voltage cannot change instantly. (If the initial condition is zero then the capacitor will look like a short just after $t = 0$.) Use normal circuit analysis to find your desired variable: $v_X(0)$ or $i_X(0)$

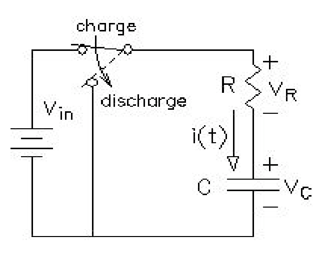
Find the final condition ("steady-state" or "forced" solution)

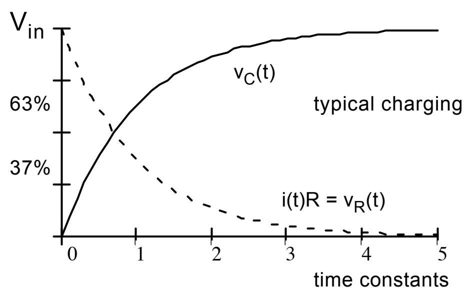
Capacitors are opens. Solve by DC analysis to find $v_X(\infty)$ or $i_X(\infty)$

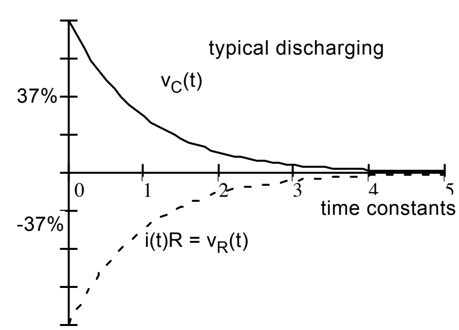
RC Time constant = $\tau = R_{Th} \cdot C$

Curves

Sample circuit

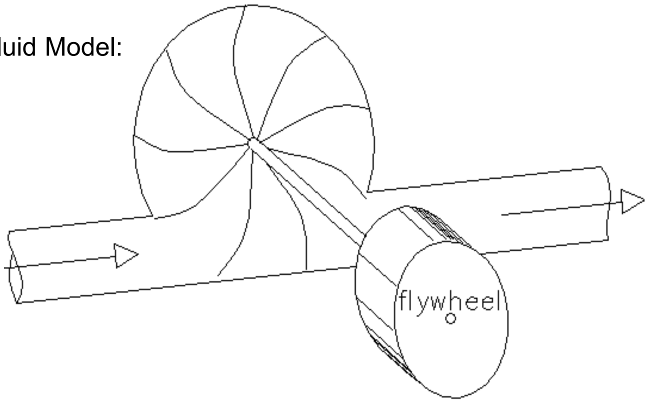




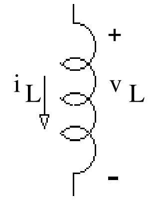


ECE 1250 Inductor Lecture Notes

Fluid Model:



Electrical equivalent:



$$L = \mu_o \cdot N^2 \cdot K$$

μ is the permeability of the inductor core

K is a constant which depends on the inductor geometry

N is the number of turns of wire

Basic equations you should know: $$v_L = L \cdot \frac{d}{dt} i_L$$

$$i_L = \frac{1}{L} \cdot \int_{-\infty}^{t} v_L \, dt$$

Or... $$i_L = \frac{1}{L} \cdot \int_{0}^{t} v_L \, dt + \underbrace{i_L(0)}_{\text{initial current}}$$

Or... $$\Delta i_L = \frac{1}{L} \cdot \int_{t_1}^{t_2} v_L \, dt$$

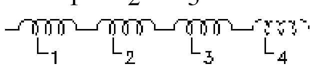
Energy stored in electric field: $W_L = \frac{1}{2} \cdot L \cdot I_L^{\,2}$

Inductor current **cannot** change instantaneously

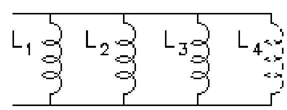
Units: $\text{henry} = \frac{\text{volt} \cdot \text{sec}}{\text{amp}}$

$\text{mH} = 10^{-3} \cdot \text{H}$ $\qquad$ $\mu\text{H} = 10^{-6} \cdot \text{H}$

series: $L_{eq} = L_1 + L_2 + L_3 + \ldots$



parallel: $$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots}$$



Sinusoids $\quad i_L(t) = I_p \cdot \cos(\omega \cdot t)$

$$v_L(t) = L \cdot \frac{d}{dt} i_L = \underbrace{L \cdot \omega}_{V_p} \cdot \left(-I_p \cdot \sin(\omega \cdot t)\right) = \underbrace{L \cdot \omega \cdot I_p}_{V_p} \cdot \cos(\omega \cdot t + 90 \cdot \text{deg})$$

Voltage "leads" current, makes sense, voltage has to present to make current change, so voltage comes first.