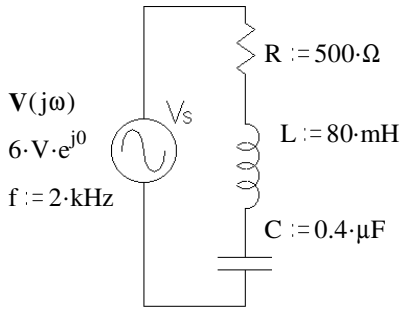


Phasor Examples

Ex 1. Find V_R , V_L , and V_C in polar phasor form. $f := 2 \cdot \text{kHz}$

A.Stolp
rev,



$$\omega := 2 \cdot \pi \cdot f \quad \omega = 1.257 \cdot 10^4 \cdot \frac{\text{rad}}{\text{sec}}$$

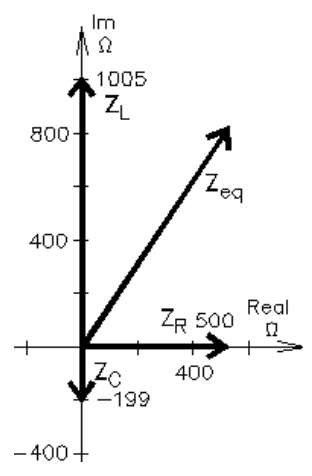
$$Z_L := j \cdot \omega \cdot L \quad Z_L = 1.005j \cdot \text{k}\Omega$$

$$Z_C := \frac{1}{j \cdot \omega \cdot C} \quad Z_C = -0.199j \cdot \text{k}\Omega$$

$$Z_{eq} := R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} \quad Z_{eq} = 500 + 806.366j \cdot \Omega$$

$$\sqrt{500^2 + 806^2} = 948.491$$

$$\text{atan}\left(\frac{806}{500}\right) = 58.187 \cdot \text{deg} \quad Z_{eq} = 948.5 \Omega / 58.2^\circ$$



find the current: $I := \frac{6 \cdot V \cdot e^{j0}}{Z_{eq}}$

magnitude: $\frac{6 \cdot V}{948.5 \Omega} = 6.326 \cdot \text{mA}$

angle: $0 \cdot \text{deg} - 58.2 \cdot \text{deg} = -58.2 \cdot \text{deg}$

$I = 6.326 \text{mA} / -58.2^\circ$

find the magnitude

find the angle

$V_R := I \cdot R \quad 6.326 \cdot \text{mA} \cdot 500 \cdot \Omega = 3.163 \cdot V$

$-58.2 \cdot \text{deg} + 0 \cdot \text{deg} = -58.2 \cdot \text{deg}$

$V_R = 3.163V / -58.2^\circ$

$V_L := I \cdot Z_L \quad 6.326 \cdot \text{mA} \cdot 1005 \cdot \Omega = 6.358 \cdot V$

$-58.2 \cdot \text{deg} + 90 \cdot \text{deg} = 31.8 \cdot \text{deg}$

$V_L = 6.358V / 31.8^\circ$

$V_C := I \cdot Z_C \quad 6.326 \cdot \text{mA} \cdot (-199) \cdot \Omega = -1.259 \cdot V$

$-58.2 \cdot \text{deg} + (90) \cdot \text{deg} = 31.8 \cdot \text{deg}$

$V_C = -1.259V / 31.8^\circ$

OR: $6.326 \cdot \text{mA} \cdot (199) \cdot \Omega = 1.259 \cdot V$

$-58.2 \cdot \text{deg} + (-90) \cdot \text{deg} = -148.2 \cdot \text{deg}$

$V_C = 1.259V / -148.2^\circ$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find V_C directly:

$$V_C := \frac{\frac{1}{j \cdot \omega \cdot C}}{R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) + j \cdot \omega \cdot L \cdot (j \cdot \omega \cdot C) + 1} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot V$$

$$= \frac{1}{(1 - \omega^2 \cdot L \cdot C) + j \cdot \omega \cdot R \cdot C} \cdot 6 \cdot V \quad (1 - \omega^2 \cdot L \cdot C) = -4.053 \quad j \cdot \omega \cdot R \cdot C = 2.513j$$

$$= \frac{6 \cdot V \cdot (-4.053 - 2.513j)}{-4.053 + 2.513j \cdot (-4.053 - 2.513j)} = \frac{6 \cdot V \cdot (-4.053 - 2.513j)}{(-4.053)^2 + 2.513^2}$$

$$6 \cdot V \cdot (-4.053 - 2.513j) = -24.318 - 15.078j \cdot V$$

$$(-4.053)^2 + 2.513^2 = 22.742$$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078j}{22.742} \right) \cdot V = -1.069 - 0.663j \cdot V$$

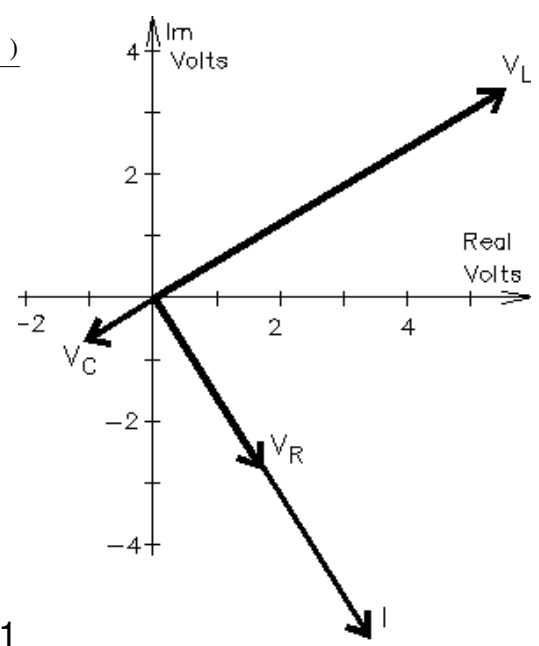
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

angle: $\text{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81 \cdot \text{deg}$

but this is actually in the third quadrant, so modify your calculator's results:

$31.81 \cdot \text{deg} - 180 \cdot \text{deg} = -148.19 \cdot \text{deg}$

$= 1.258V / -148.2^\circ$



Phasor Examples p2

Ex 2. a) Find Z_{eq} . $f := 2.5 \cdot \text{kHz}$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.571 \cdot 10^4 \frac{\text{rad}}{\text{sec}}$

$$Z_{eq} = j \cdot \omega \cdot L_1 + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right) + \frac{1}{R + j \cdot \omega \cdot L_2}}$$

But it's easier to split the problem up

Left branch

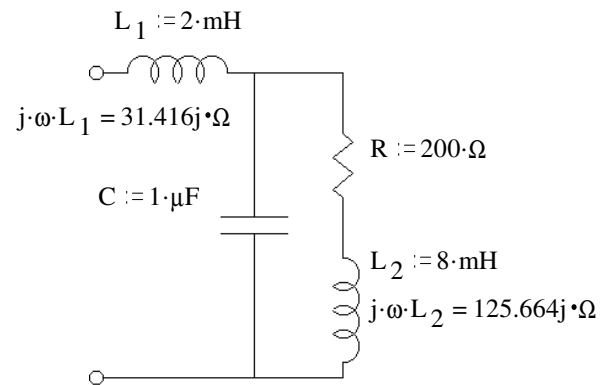
$$Z_1 := \frac{1}{j \cdot \omega \cdot C} \quad Z_1 = -63.662j \cdot \Omega$$

$$\frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} = j \cdot \omega \cdot C = 0.01571i \cdot \frac{1}{\Omega}$$

Right branch

$$Z_R := j \cdot \omega \cdot L_2 + R \quad Z_R = 200 + 125.664j \cdot \Omega$$

$$\frac{1}{200 + 125.664j} = 3.585 \cdot 10^{-3} - 2.252 \cdot 10^{-3}j$$



denominator: $j \cdot \omega \cdot C + \frac{1}{R + j \cdot \omega \cdot L_2} = 0.01571j + (3.585 \cdot 10^{-3} - 2.252 \cdot 10^{-3}j) = 3.585 \cdot 10^{-3} + 1.346 \cdot 10^{-2}i \quad \frac{1}{\Omega}$

rectangular division:

$$\frac{1}{(3.585 \cdot 10^{-3} + 1.346 \cdot 10^{-2}j)} \cdot \frac{(3.585 \cdot 10^{-3} - 1.346 \cdot 10^{-2}j)}{(3.585 \cdot 10^{-3} - 1.346 \cdot 10^{-2}j)} = \frac{3.585 \cdot 10^{-3} - 1.346 \cdot 10^{-2}j}{1.94 \cdot 10^{-4}} = 18.479 - 69.381j \quad \Omega$$

$$(3.585 \cdot 10^{-3})^2 + (1.346 \cdot 10^{-2})^2 = 1.94 \cdot 10^{-4}$$

add: $j \cdot \omega \cdot L_1 = 31.416j \cdot \Omega$ $31.416j + (18.479 - 69.381j) = 18.479 - 37.965j \quad \Omega$

convert to polar (if needed): $\sqrt{18.48^2 + 37.97^2} = 42.228$ $\text{atan}\left(\frac{-37.97}{18.48}\right) = -64.048 \cdot \text{deg}$ $Z_{eq} = 42.23 \Omega \angle -64.05^\circ$

Another Way

Sometimes you might simplify a little before putting in numbers.

$$Z_{eq} := j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + \frac{1}{j \cdot \omega \cdot C}} = j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C} = j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 + j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}$$

$$= j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

$$Z_{eq} = 31.416j \cdot \Omega + \frac{(200 + 125.664j) \cdot \Omega}{-0.974 + 3.142j} \cdot \frac{(-0.974 - 3.142j)}{(-0.974 - 3.142j)} = 31.416j \cdot \Omega + \frac{(200 + 125.664j) \cdot (-0.974 - 3.142j)}{0.974^2 + 3.142^2}$$

$$= 31.416j \cdot \Omega + \frac{((200 \cdot (-0.974) - 125.664 \cdot (-3.142)) + (125.664 \cdot (-0.974) - 200 \cdot 3.142)j) \cdot \Omega}{0.974^2 + 3.142^2}$$

$$= 31.416j \cdot \Omega + \frac{(200.036288 - 750.796736j) \cdot \Omega}{10.82084} = 31.416j \cdot \Omega + 18.486 \cdot \Omega - 69.384j \cdot \Omega = 18.486 - 37.968j \cdot \Omega$$

$$\sqrt{18.49^2 + 37.97^2} = 42.233 \quad \text{atan}\left(\frac{-37.97}{18.49}\right) = -64.036 \cdot \text{deg} \quad Z_{eq} = 42.23 \Omega \angle -64.04^\circ$$

Phasor Examples p2

a little roundoff difference

b) $V_{in} := 12 \cdot V \cdot e^{j20 \cdot \text{deg}}$ Find I_{L1} , V_C $I_{L1} := \frac{V_{in}}{Z_{eq}} = \frac{12 \cdot V}{42.23 \cdot \Omega} = 284.16 \cdot \text{mA}$ $20 \cdot \text{deg} - (-64.04) \cdot \text{deg} = 84.04 \cdot \text{deg}$

$I_{L1} = 284 \text{mA} / 84.04^\circ$

$V_C := I_{L1} \cdot (18.479 - 69.381 \cdot j) \cdot \Omega = 284 \cdot \text{mA} \cdot \sqrt{18.479^2 + 69.381^2} \cdot \Omega = 20.391 \cdot V$

$84.04 \cdot \text{deg} + \text{atan}\left(\frac{-69.381}{18.479}\right) = 8.954 \cdot \text{deg}$

$V_C = 20.4 \text{V} / 8.95^\circ$

You could then use another voltage divider to find V_R or V_{L2} .

convert to rectangular (if needed): $20.391 \cdot V \cdot \cos(8.954 \cdot \text{deg}) = 20.143 \cdot V$

$20.391 \cdot V \cdot \sin(8.954 \cdot \text{deg}) = 3.174 \cdot V$

$V_C = 20.14 + 3.174 \cdot j \text{ V}$

Another Way

To find V_C

directly:

$$V_C := \frac{\frac{1}{R + j \cdot \omega \cdot L_2}}{j \cdot \omega \cdot L_1 + \frac{1}{R + j \cdot \omega \cdot L_2}} \cdot V_{in}$$

--> math --> $V_C = 20.153 + 3.178j \cdot V$ Same but for a little roundoff difference

c) Let's find I_{L2} . $Z_r = 200 + 125.664j \cdot \Omega$ $\sqrt{200^2 + 125.664^2} = 236.202$ $\text{atan}\left(\frac{125.664}{200}\right) = 32.142 \cdot \text{deg}$

$I_{L2} := \frac{V_C}{Z_r} = \frac{20.4 \cdot V \cdot e^{j8.95 \cdot \text{deg}}}{236.202 \cdot \Omega \cdot e^{j32.142 \cdot \text{deg}}} = \frac{20.4 \cdot V}{236.202 \cdot \Omega} / 8.95 - 32.142^\circ = 86.4 \text{mA} / -23.19^\circ$

Another Way

Directly by

Current divider:

$$I_{L2} := \frac{\frac{1}{R + j \cdot \omega \cdot L_2}}{j \cdot \omega \cdot C + \frac{1}{R + j \cdot \omega \cdot L_2}} \cdot I_{L1} = \frac{1}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot I_{L1} = \frac{I_{L1}}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

denominator:

$\sqrt{(1 - \omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.289$ $\text{atan}\left(\frac{\omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2}\right) + 180 \cdot \text{deg} = 107.224 \cdot \text{deg}$ real part is negative

$I_{L2} = \frac{284 \cdot \text{mA} \cdot e^{j84.04 \cdot \text{deg}}}{3.289 \cdot e^{j107.224 \cdot \text{deg}}} = \frac{284 \cdot \text{mA}}{3.289} / 84.04 - 107.224^\circ = 86.4 \text{mA} / -23.18^\circ$

d) How about I_C ? $I_C := \frac{V_C}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} = V_C \cdot j \cdot \omega \cdot C = 20.4 \text{V} / 8.95^\circ \cdot 0.015708 / 90^\circ \cdot \frac{1}{\Omega} = 320 \text{mA} / 98.95^\circ$

Another Way Could also be found directly by current divider: $I_C := \frac{j \cdot \omega \cdot C}{j \cdot \omega \cdot C + \frac{1}{R + j \cdot \omega \cdot L_2}} \cdot I_{L1} = 320 \text{mA} / 98.95^\circ$

Something Weird

I_C is greater than the input current (I_{L1}) . What's going on?

The angle between I_C & I_{L2} is big enough that they somewhat cancel each other out (partially resonate).

Check Kirchoff's Current Law: $I_C + I_{L2} = 29.485 + 282.569j \cdot \text{mA} = I_{L1} = 29.485 + 282.569j \cdot \text{mA}$

Phasor Examples p4

Ex 3. a) Find Z_2 .

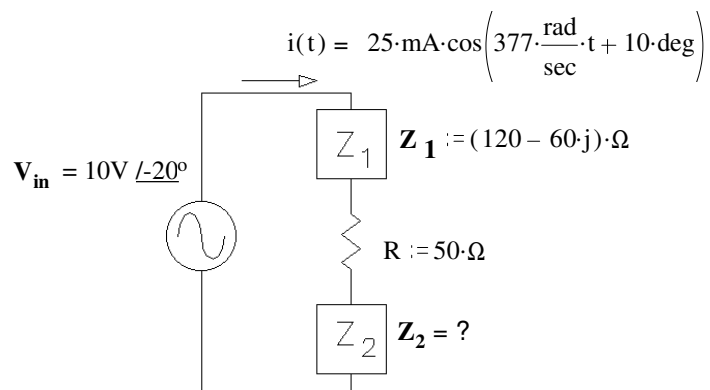
$$I := 25 \cdot \text{mA} \cdot e^{j10 \text{ deg}}$$

$$V_{in} := 10 \cdot \text{V} \cdot e^{-j20 \text{ deg}}$$

$$Z_T := \frac{V_{in}}{I} = \frac{10 \cdot \text{V}}{25 \cdot \text{mA}} \angle -20 - 10 = 400 \Omega \angle -30^\circ$$

$$Z_T = 346.41 - 200j \cdot \Omega$$

$$Z_2 := Z_T - R - Z_1 = (346.41 - 200j) \cdot \Omega - 50 \cdot \Omega - (120 - 60j) \cdot \Omega = 176.41 - 140j \cdot \Omega$$



- b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^\circ > -20^\circ$
 ii) The source voltage leads the source current

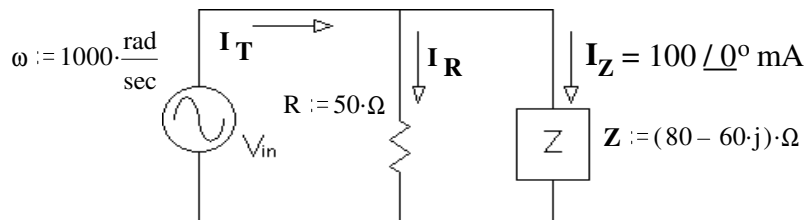
Ex 4. a) Find V_{in} in polar form.

$$I_Z := 100 \cdot \text{mA} \quad Z := (80 - 60j) \cdot \Omega$$

$$V_{in} := I_Z \cdot Z \quad V_{in} = 8 - 6j \cdot \text{V}$$

$$\sqrt{8^2 + 6^2} = 10 \quad \text{atan}\left(\frac{-6}{8}\right) = -36.87 \cdot \text{deg}$$

$$V_{in} = 10 \text{V} \angle -36.9^\circ$$



b) Find I_T in polar form. $I_R := \frac{V_{in}}{R} = \frac{10 \cdot \text{V}}{50 \cdot \Omega} \angle -36.9^\circ = \frac{10 \cdot \text{V}}{50 \cdot \Omega} \cdot \cos(-36.9 \cdot \text{deg}) + j \cdot \frac{10 \cdot \text{V}}{50 \cdot \Omega} \cdot \sin(-36.9 \cdot \text{deg}) = 160 - 120j \cdot \text{mA}$

$$I_T := I_R + I_Z = (160 - 120j) \cdot \text{mA} + 100 \cdot \text{mA} = 260 - 120j \cdot \text{mA}$$

$$\sqrt{260^2 + 120^2} = 286.356 \quad \text{atan}\left(\frac{-120}{260}\right) = -24.78 \cdot \text{deg} \quad I_T = 286 \text{mA} \angle -24.8^\circ$$

- c) Circle 1: i) The source current leads the source voltage answer i), $-24.8^\circ > -36.9^\circ$
 ii) The source voltage leads the source current

d) The impedance Z (above) is made of two components in series. What are they and what are their values?

$$Z = 80 - 60j \cdot \Omega$$

Must have a resistor because there is a real part.

$$R := \text{Re}(Z)$$

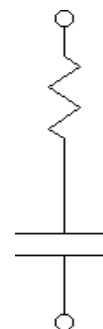
$$R = 80 \cdot \Omega$$

Must have a capacitor because the imaginary part is negative.

$$\text{Im}(Z) = -60 \cdot \Omega = \frac{-1}{\omega \cdot C}$$

$$C := \frac{-1}{\omega \cdot \text{Im}(Z)}$$

$$C = 16.667 \cdot \mu\text{F}$$



Phasor Examples p5

Ex 5. The impedance $Z = 80 - 60j \cdot \Omega$ is made of two components in parallel. What are they and what are their values?

Must have a resistor because there is a real part.

Must have an capacitor because the imaginary part is negative.

$$Z = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C} \quad \frac{1}{Z} = \frac{1}{(80 - 60j) \cdot \Omega} \cdot \frac{(80 + 60j)}{(80 + 60j)} = \frac{80 + 60j}{80^2 + 60^2} = \frac{80 + 60j}{10,000} \cdot \frac{1}{\Omega}$$

$$\frac{1}{Z} = 0.008 + 0.006j \cdot \Omega^{-1} = \frac{1}{R} + j \cdot \omega \cdot C$$

$$\frac{1}{R} = .008 \cdot \frac{1}{\Omega}$$

$$R := \frac{1}{.008 \cdot \Omega^{-1}}$$

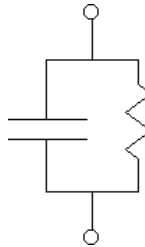
$$R = 125 \cdot \Omega$$

$$\omega \cdot C = .006 \cdot \frac{1}{\Omega}$$

$$C := \frac{.006 \cdot \Omega^{-1}}{\omega}$$

$$C = 6 \cdot \mu\text{F}$$

$$R = 125 \cdot \Omega$$



Positive imaginary parts would require inductors

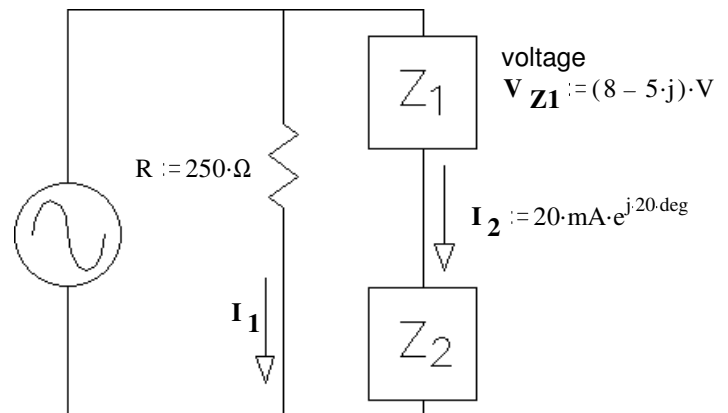
Ex 6. a) Find I_1

$$\omega := 20000 \cdot \frac{\text{rad}}{\text{sec}}$$

$$V_{in} := 20 \cdot V \cdot e^{j30 \cdot \text{deg}}$$

$$I_1 := \frac{V_{in}}{R} = \frac{20 \cdot V}{250 \cdot \Omega} \cdot e^{j30 \cdot \text{deg}} = 80 \cdot \text{mA} \cdot e^{j30 \cdot \text{deg}}$$

polar division



b) Circle 1:

i) V_{in} leads I_2

ii) V_{in} lags I_2

Why? Show numbers:

$$\underline{30} > \underline{20}$$

$$\underline{\quad} < \underline{\quad}$$

c) Find Z_2 in polar form

Convert V_{in} to rectangular coordinates

$$20 \cdot V \cdot \cos(30 \cdot \text{deg}) = 17.321 \cdot V$$

$$20 \cdot V \cdot \sin(30 \cdot \text{deg}) = 10 \cdot V$$

pol to rect

$$V_{in} = 17.321 + 10j \cdot V$$

$$V_{Z2} := V_{in} - V_{Z1}$$

$$V_{Z2} = 9.321 + 15j \cdot V$$

subtract

$$\text{rect to pol} \quad \sqrt{9.321^2 + 15^2} = |V_{Z2}| = 17.66 \cdot V$$

$$\text{atan}\left(\frac{15}{9.321}\right) = \arg(V_{Z2}) = 58.145 \cdot \text{deg}$$

$$\text{div} \quad Z_2 := \frac{V_{Z2}}{I_2} = \frac{17.66 \cdot V}{20 \cdot \text{mA}} = 883 \cdot \Omega$$

$$\angle 58.145 \cdot \text{deg} - 20 \cdot \text{deg} = 38.145 \cdot \text{deg}$$

$$Z_2 = 883 / 38.15^\circ \Omega$$

$$Z_2 = 694.436 + 545.379j \cdot \Omega$$

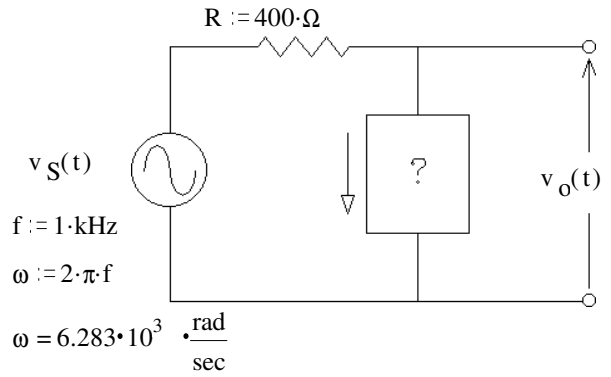
Ex 7. You need to design a circuit in which the "output" voltage leads the input voltage ($v_S(t)$) by 40° of phase.

a) What should go in the box: R, L, C?

$$V_o = \frac{Z_{\text{box}}}{R + Z_{\text{box}}} \cdot V_S$$

angle of $\frac{Z_{\text{box}}}{R + Z_{\text{box}}}$ is 40° .

This can only happen if the angle of Z_{box} is positive, so Z_{box} is an inductor



b) Find its value. $V_o = \frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L} \cdot V_S$ angle $\frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L}$ is $90 - \text{atan}\left(\frac{\omega \cdot L}{R}\right) = 40^\circ$.

So: $\text{atan}\left(\frac{\omega \cdot L}{R}\right) = 50^\circ$ $\frac{\omega \cdot L}{R} = \tan(50 \cdot \text{deg}) = 1.192$ $L = \frac{R \cdot 1.192}{\omega} = 75.9 \cdot \text{mH}$

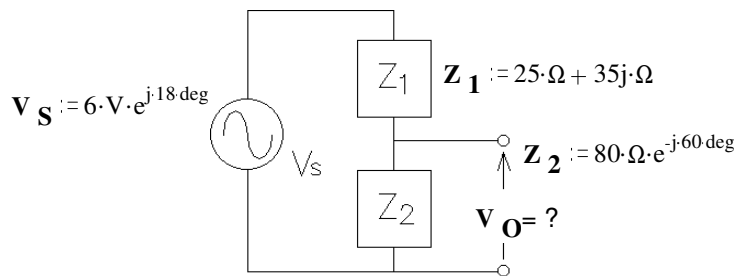
c) Repeat if the "output" voltage should lag the input voltage ($v_S(t)$) by 20° of phase.

angle of $\frac{Z_{\text{box}}}{R + Z_{\text{box}}}$ is -20° . This can only happen if the angle of Z_{box} is negative, so Z_{box} is a capacitor

$$V_o = \frac{1}{R + \frac{1}{j \cdot \omega \cdot C}} \cdot V_S \quad \text{angle} \quad \frac{1}{R + \frac{1}{j \cdot \omega \cdot C}} \quad \text{is} \quad -90 - \text{atan}\left(\frac{1}{\omega \cdot C \cdot R}\right) = -90 - \text{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right)$$

$\text{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right) = -70^\circ$ $-\frac{1}{\omega \cdot C \cdot R} = \tan(-70 \cdot \text{deg}) = -2.747$ $C = \frac{1}{\omega \cdot R \cdot 2.747} = 0.145 \cdot \mu\text{F}$

Ex 8. Find V_o in the circuit shown. Express it as a magnitude and phase angle (polar).



$V_o := \frac{Z_2}{Z_1 + Z_2} \cdot V_S$ Simple voltage divider

$|Z_2| \cdot \cos(-60 \cdot \text{deg}) = 40 \cdot \Omega$ $|Z_2| \cdot \sin(-60 \cdot \text{deg}) = -69.282 \cdot \Omega$ $Z_2 = 40 - 69.282j \cdot \Omega$

$Z_1 + Z_2 = 25 \cdot \Omega + 35j \cdot \Omega + 40 \cdot \Omega - 69.282j \cdot \Omega = 65 - 34.282j \cdot \Omega = 73.486 \cdot \Omega \cdot e^{-j27.81 \cdot \text{deg}}$

$V_o := \frac{Z_2}{Z_1 + Z_2} \cdot V_S = \frac{80 \cdot \Omega \cdot e^{-j60 \cdot \text{deg}}}{73.486 \cdot \Omega \cdot e^{-j27.81 \cdot \text{deg}}} \cdot (6 \cdot V \cdot e^{j18 \cdot \text{deg}}) = \frac{80 \cdot \Omega}{73.486 \cdot \Omega} \cdot 6 \cdot V \cdot e^{j(-60 - (-27.81) + 18) \cdot \text{deg}} = 6.53 \cdot V \cdot e^{-j14.2 \cdot \text{deg}}$