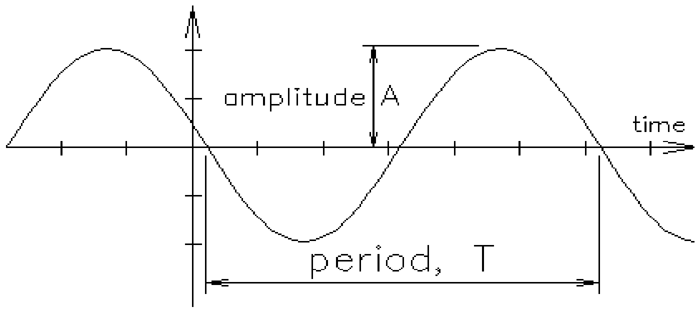
ECE 1250

Introduction to AC Phasors

Phasor analysis with impedances, For steady-state sinusoidal response ONLY

Sinusoidal AC



T = Period = repeat time

f = frequency, cycles / second $f = \frac{1}{T} = \frac{\omega}{2\cdot\pi}$

ω = radian frequency, radians/sec $\omega = 2\cdot\pi\cdot f$

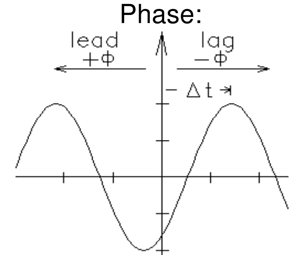
A = amplitude

Phase: $\phi = -\frac{\Delta t}{T}\cdot 360\cdot deg$

or: $\phi = -\frac{\Delta t}{T}\cdot 2\cdot\pi\cdot rad$

$y(t) = A\cdot\cos(\omega\cdot t + \theta)$

Phase:



Phasor analysis

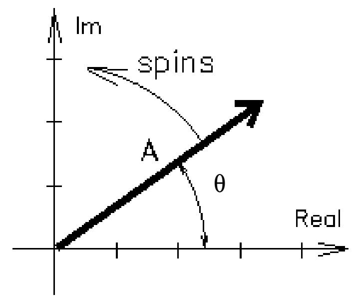
The math is all based on the Euler's equation

Euler's equation $e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$

OR:

$\cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2}$

$\sin(\theta) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$



$e^{j\cdot(\omega\cdot t+\theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$

$Re\left[e^{j\cdot(\omega\cdot t+\theta)}\right] = \cos(\omega\cdot t + \theta)$

If we freeze this at time $t=0$, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$ That's the phasor

Phasor

voltage: $v(t) = V_p\cdot\cos(\omega\cdot t + \phi)$ $\mathbf{V}(\omega) = V_p\cdot e^{j\cdot\phi}$

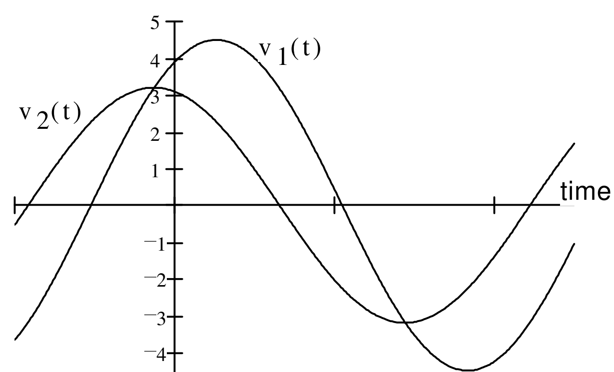
current: $i(t) = I_p\cdot\cos(\omega\cdot t + \phi)$ $\mathbf{I}(\omega) = I_p\cdot e^{j\cdot\phi}$

Phasors are drawn on a complex plane.

Phasors are used for adding and subtracting sinusoidal waveforms.

Ex1. Add the sinusoidal voltages $v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

and $v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$



using phasor notation, draw a phasor diagram of the three phasors, then convert back to time domain form.

$v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

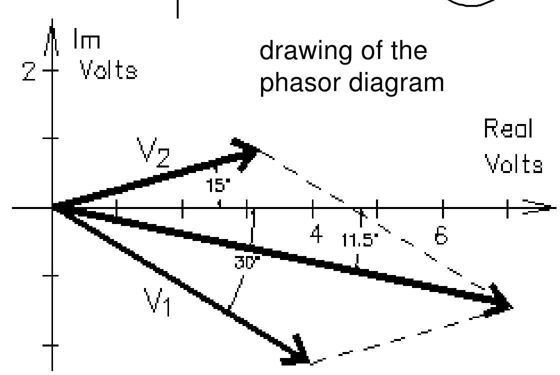
$\mathbf{V_1}(\omega) = 4.5V \angle{-30^o}$ or: $\mathbf{V_1}(\omega) = 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

and

$v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

$\mathbf{V_2}(\omega) = 3.2V \angle{15^o}$ or: $\mathbf{V_2}(\omega) = 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

drawing of the phasor diagram



I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$\mathbf{V_1} = 4.5V \angle{-30^o}$ or: $\mathbf{V_1} := 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

$\mathbf{V_2} = 3.2V \angle{15^o}$ or: $\mathbf{V_2} := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

Add like vectors, first change to the rectangular form

$\mathbf{V_1} = 4.5\text{V} \underline{/-30^o}$ $\quad 4.5\cdot V\cdot\cos(-30\cdot deg) = 3.897\cdot V$ $\quad 4.5\cdot V\cdot\sin(-30\cdot deg) = -2.25\cdot V$ $\quad \mathbf{V_1} = 3.897 - 2.25j \cdot V$ \

} add

$\mathbf{V_2} = 3.2\text{V} \underline{/15^o}$ $\quad 3.2\cdot V\cdot\cos(15\cdot deg) = 3.091\cdot V$ $\quad 3.2\cdot V\cdot\sin(15\cdot deg) = 0.828\cdot V$ $\quad \mathbf{V_2} = 3.091 + 0.828j \cdot V$ /

$\mathbf{V_3} := \mathbf{V_1} + \mathbf{V_2}$

Add real parts: $3.897 + 3.091 = 6.988$

Add imaginary parts: $-2.25 + 0.828 = -1.422$ $\quad \mathbf{V_3} = 6.988 - 1.422j \cdot V$ sum

Change $\mathbf{V_3}$ back to polar coordinates:

$\sqrt{6.988^2 + 1.422^2} = 7.131$ $\qquad \operatorname{atan}\left(\frac{-1.422}{6.988}\right) = -11.502\cdot deg$

OR, in Mathcad notation (you' ll see these in future solutions):

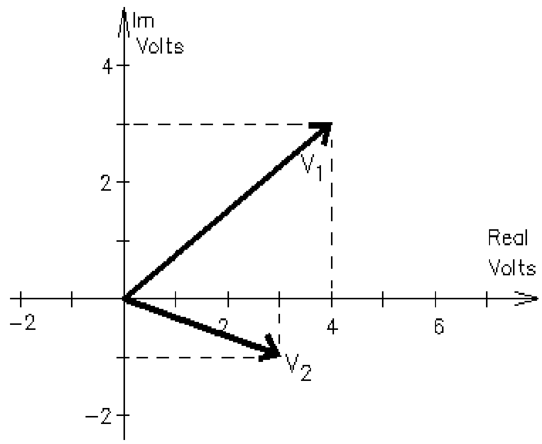
$|\mathbf{V_3}| = 7.131\cdot V$ $\qquad \arg(\mathbf{V_3}) = -11.5\cdot deg$

Change $\mathbf{V_3}$ back to the time domain:

$v_3(t) = v_1(t) + v_2(t) = 7.13\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$

Ex 2. Two sinusoidal voltages: $v_1(t) = 5\cdot V\cdot\cos(\omega\cdot t + 36.87\cdot deg)$ and $v_2(t) = 3.162\cdot V\cdot\cos(\omega\cdot t - 18.44\cdot deg)$

a) using phasor notation, find $v_3 = v_1 - v_2$



$\mathbf{V_1} := 5\cdot V\cdot e^{j\cdot(36.87\cdot deg)}$ $\qquad 5\cdot V\cdot\cos(36.87\cdot deg) = 4\cdot V$

$5\cdot V\cdot\sin(36.87\cdot deg) = 3\cdot V$

$\mathbf{V_1} = 4 + 3j \cdot V$

$\mathbf{V_2} := 3.162\cdot V\cdot e^{j\cdot(-18.44\cdot deg)}$ $\qquad 3.162\cdot V\cdot\cos(-18.44\cdot deg) = 3\cdot V$

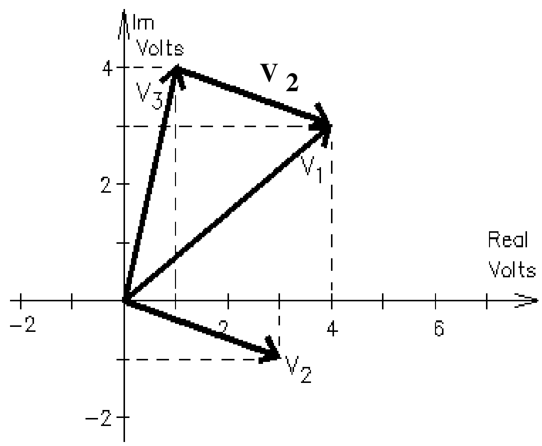
$3.162\cdot V\cdot\sin(-18.44\cdot deg) = -1\cdot V$

$\mathbf{V_2} = 3 - j \cdot V$

Subtract real parts: $4\cdot V - 3\cdot V = 1\cdot V$

Subtract imaginary parts: $3\cdot V - -1\cdot V = 4\cdot V$

$\mathbf{V_3} := \mathbf{V_1} - \mathbf{V_2}$ $\qquad \mathbf{V_3} = 1 + 4j \cdot V$



Magnitude: $\sqrt{(1\cdot V)^2 + (4\cdot V)^2} = 4.123\cdot V$ $\qquad$ OR: $|\mathbf{V_3}| = 4.123\cdot V$

Angle: $\operatorname{atan}\left(\frac{4\cdot V}{1\cdot V}\right) = 75.96\cdot deg$ $\qquad \arg(\mathbf{V_3}) = 75.96\cdot deg$

So: $v_3(t) = v_1(t) - v_2(t) = 4.123\cdot V\cdot\cos(\omega\cdot t + 75.96\cdot deg)\cdot V$

What about Capacitors and Inductors?

Capacitors and Inductors in AC circuits cause 90^o phase shifts between voltages and currents because they integrate and differentiate. But... integration and differentiation is a piece-of-cake in phasors.

Intro to Phasors p2

Calculus

$$\frac{d}{dt}\left[A\cdot e^{j\cdot(\omega\cdot t+\theta)}\right] = j\cdot\omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta+90\cdot deg)} = \omega\cdot A\cdot e^{j\cdot(\theta+90\cdot deg)}$$

Drop the ωt (t=0) to get:

$$\int A\cdot e^{j\cdot(\omega\cdot t+\theta)}\,dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta-90\cdot deg)} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta-90\cdot deg)}$$

Impedance (like resistance)

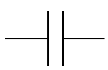
AC impedance

Inductor

$$v_L = L\cdot\frac{d}{dt}i_L = L\cdot\frac{d}{dt}I_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot L\cdot\left[I_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

\ in phasor notation ----> $\mathbf{V_L}(\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(\omega)$ $\qquad \mathbf{Z_L} = j\cdot\omega\cdot L$

Capacitor



$$i_C = C\cdot\frac{d}{dt}v_C = C\cdot\frac{d}{dt}V_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot C\cdot\left[V_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

\ in phasor notation ----> $\mathbf{I_C}(\omega) = j\cdot\omega\cdot C\cdot\mathbf{V}(\omega)$

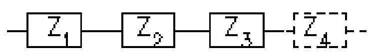
$$\mathbf{V_C}(\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(\omega) \qquad \mathbf{Z_C} = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$$

Resistor

$$v_R = i_R\cdot R \qquad \mathbf{V_R}(\omega) = R\cdot\mathbf{I}(\omega) \qquad \mathbf{Z_R} = R$$

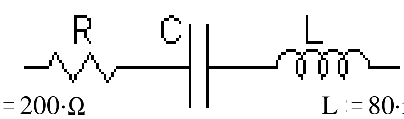
You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:



$$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$$

Example:



$f := 500\cdot Hz$ $\qquad R := 200\cdot\Omega \qquad C := 0.6\cdot\mu F \qquad L := 80\cdot mH$

$$\omega := 2\cdot\pi\cdot f = \omega = 3141.6\cdot\frac{rad}{sec}$$

$$j\cdot\omega\cdot L = 251.327j\cdot\Omega \qquad \frac{1}{j\cdot\omega\cdot C} = -530.516j\cdot\Omega$$

$$\mathbf{Z_{eq}} := R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L = 200\cdot\Omega - 530.5\cdot j\cdot\Omega + 251.3\cdot j\cdot\Omega = 200 - 279.2j\cdot\Omega$$ rectangular form

$$\sqrt{(200\cdot\Omega)^2 + (279.2\cdot\Omega)^2} = 343.4\cdot\Omega \qquad \operatorname{atan}\left(\frac{-279.2\cdot\Omega}{200\cdot\Omega}\right) = -54.38\cdot deg$$

$\mathbf{Z_{eq}} = 343.4\Omega\ \underline{/-54.4}^{o}$ polar form

If: $\mathbf{V} := 12\cdot V\cdot e^{j\cdot 0\cdot deg}$ $\qquad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z_{eq}}} = \frac{12\cdot V}{343.4\cdot\Omega} = 34.945\cdot mA \quad \underline{/}\ 0 - -54.4 = 54.4$ deg

$\mathbf{I} = 34.95mA\ \underline{/54.4}^{o} = \mathbf{I} = 20.348 + 28.405j\cdot mA$

Voltage divider:

$$\mathbf{V_{Zn}} = \mathbf{V_{total}}\cdot\frac{\mathbf{Z_n}}{\mathbf{Z_1}+\mathbf{Z_2}+\mathbf{Z_3}} + \ldots$$

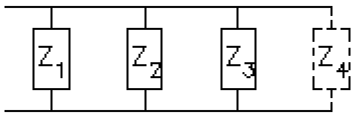
Note: $\frac{1}{j} = -j = 1\ \underline{/-90}^{o}$

Eg: $\mathbf{V_C} := \mathbf{V}\cdot\frac{\frac{1}{j\cdot\omega\cdot C}}{\mathbf{Z_{eq}}} = 12\cdot V\cdot e^{j\cdot 0\cdot deg}\cdot\frac{530.516\cdot e^{-j\cdot 90\cdot deg}\cdot\Omega}{343.4\cdot e^{-j\cdot 54.38\cdot deg}\cdot\Omega}$

$$12\cdot V\cdot\frac{530.516\cdot\Omega}{343.4\cdot\Omega} = 18.539\cdot V \quad \underline{/}\ 0 + -90 - -54.4 = -35.6 \text{ deg}$$

$\mathbf{V_C} = 18.54V\ \underline{/-35.6}^{o} = \mathbf{V_C} = 15.069 - 10.795j\cdot V$

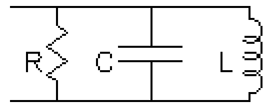
parallel:



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Example:

$f := 500 \cdot Hz \qquad \omega := 2 \cdot \pi \cdot f \qquad \omega = 3141.6 \cdot \frac{rad}{sec}$



$L := 80 \cdot mH$

$R := 200 \cdot \Omega \qquad C := 0.6 \cdot \mu F$

$j \cdot \omega \cdot L = 251.327j \cdot \Omega$

$\frac{1}{\omega \cdot L} = 3.979 \cdot 10^{-3} \cdot \frac{1}{\Omega}$

$\frac{1}{j \cdot \omega \cdot C} = -530.516j \cdot \Omega$

$\omega \cdot C = 1.885 \cdot 10^{-3} \cdot \frac{1}{\Omega}$

$$Z_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{200 \cdot \Omega} + 1.885 \cdot 10^{-3} \cdot j \cdot \frac{1}{\Omega} - 3.979 \cdot 10^{-3} \cdot j \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-3} - 2.094 \cdot 10^{-3} \cdot j\right) \cdot \frac{1}{\Omega}} \cdot \frac{5 \cdot 10^{-3} + 2.094 \cdot 10^{-3} \cdot j}{\left(5 \cdot 10^{-3} + 2.094 \cdot 10^{-3} \cdot j\right)} = 170.156 + 71.261j \cdot \Omega$$

$2.93848 \cdot 10^{-5}$

If you want the answer in polar form, it' s easier to convert the denominator first.

$$\sqrt{\left(5 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2 + \left(2.094 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2} = 5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega} \qquad \operatorname{atan}\left(\frac{2.094 \cdot 10^{-3} \cdot \Omega}{5 \cdot 10^{-3} \cdot \Omega}\right) = 22.72 \cdot deg$$

$$\frac{1}{5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega}} = 185.185 \cdot \Omega$$

$Z_{eq} = 185.2 \angle 22.7°$

If: $\mathbf{V} := 12 \cdot V \cdot e^{j \cdot 0\,deg}$ $\qquad \mathbf{I} := \frac{\mathbf{V}}{Z_{eq}} = \frac{12 \cdot V}{185.2 \cdot \Omega} = 64.795 \cdot mA \quad \angle \; 0 - 22.7 = -22.7 \;\; deg$

$\mathbf{I} = 60 - 25.127j \cdot mA$

Current divider:

$$I_{Zn} = I_{total} \cdot \frac{\frac{1}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Eg: $\mathbf{I_L} := \mathbf{I} \cdot \frac{\frac{1}{j \cdot \omega \cdot L}}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}} = \mathbf{I} \cdot \frac{Z_{eq}}{j \cdot \omega \cdot L}$

$$= 64.795 \cdot mA \cdot e^{j \cdot 22.7 \cdot deg} \cdot \frac{185.2 \cdot e^{-j \cdot 22.7 \cdot deg} \cdot \Omega}{251.327 \cdot e^{j \cdot 90\,deg} \cdot \Omega}$$

$$= 64.795 \cdot mA \cdot \frac{185.2 \cdot \Omega}{251.327 \cdot \Omega} = 47.747 \cdot mA \quad \angle \; 22.7 + -22.7 - 90 = -90 \quad deg$$

$\mathbf{I_L} = -47.746j \cdot mA$

Duh... $\frac{\mathbf{V}}{j \cdot \omega \cdot L} = -47.746j \cdot mA$