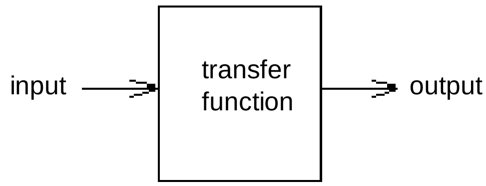
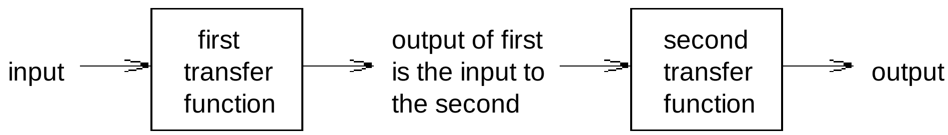
A. Stolp
2/14/12

Blocks and block diagrams (acting on signals (information))

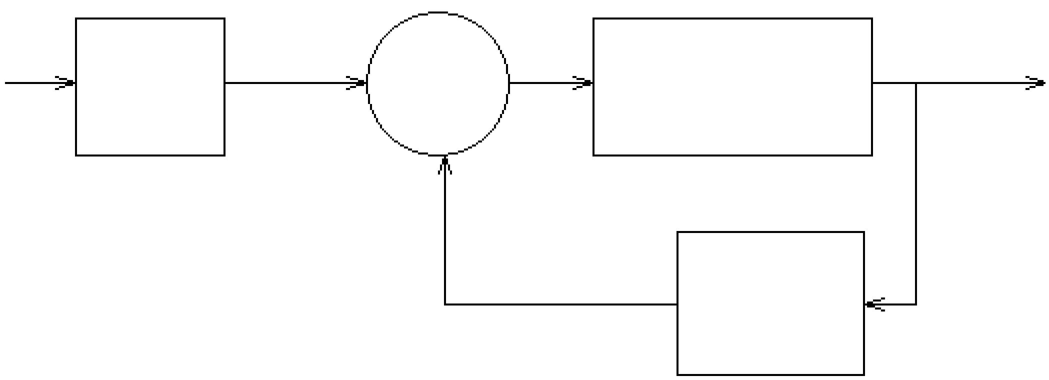


These blocks DO NOT show the flow of materials, power, or energy needed to act upon the information. For example, the input to a block might be the postion of the gas pedal in your car and the output might be the car's speed. The energy input required is not shown and niether are the fuel and air moving through the engine.

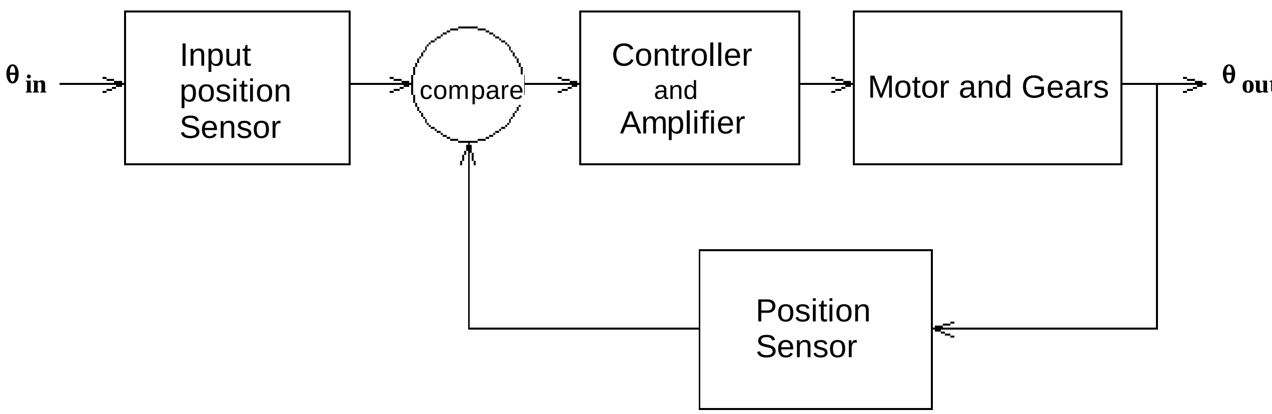
Blocks can be "hooked" together, that is, the output of one block could be the input to another



Or even in a loop, here is an example with feedback.



Example of a System (A Position Servo with Feedback)



Again, the lines represent signals. Yes, there may also be considerable power moving from one block to another or out the end, but that's not what we'll care about here. All we really care about here is the basic information.

Blocks represent subsystems, devices or components which act upon an input or inputs to produce an output.

We will want a **mathmatical** way to represent the signals and the action of the blocks so that we can get a better handle on what's happening, and, hopefully, make the whole sytem work as we want.

We'll assume that each of the blocks is linear and time invariant. Anything else gets too hard too fast, and this is a good place to start. Many real devices can be modeled as linear and time invariant, at least over some region of operation.

We'd like to work with the blocks in a very simple way:

input → [transfer function] → output = input x transfer function

In this case, the transfer function = $\frac{\text{output}}{\text{input}}$

What's inside?

How are the input and output related? If you know the input, how do you find the output? Sometimes we can just multiply the input by the expression in the box to get the output. Then the expression in the box is called a transfer function.

In that case, the transfer function = $\frac{\text{output}}{\text{input}}$

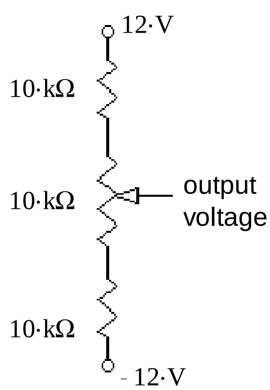
Amplfier, a very simple case that we've already seen.

input V_{in} → [Gain 22] → output $V_{out} = 22 \cdot V_{in}$

example with
a gain of 22

Of course, this only works over the limits of the amplfier. But those are non-linearities that we are neglecting now.

Another simple case, a potentiometer measuring position



the input is θ ,
the angle of the shaft

We measure the voltage over its range of motion to find Kp.

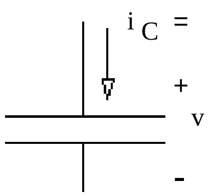
$$K_p = \frac{\text{output}}{\text{input}} = \frac{4 \cdot V - -4 \cdot V}{270 \cdot deg} = 29.63 \cdot \frac{mV}{deg} = 1.7 \cdot \frac{V}{rad}$$

range of motion

In this case, "zero" must be in the center of the range of motion

$\theta_{in}(t)$ → [$Kp = 0.7 \cdot \frac{V}{rad} = 0.012 \cdot \frac{V}{deg}$] → $V_{out}(t) = K_p \cdot \theta_{in}(t)$

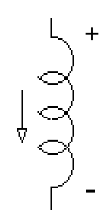
Nice... too bad it works for so few things in the time domain! Simple voltage dividers, amplifiers, and not much else. In real electrical systems there are always capacitance and inductance.



$$i_C = C \cdot \frac{d}{dt} v_C$$

$$v_C = \frac{1}{C} \cdot \int i_C \, dt$$

$$\frac{1}{L} \cdot \int v_L \, dt = i_L$$



$$v_L = L \cdot \frac{d}{dt} i_L$$

We'll have to avoid capacitors and inductors-- they're too complicated... You can't just multiply when there are differentials involved

How about the mechanical world? $F = ma$, Great, no differentials... uh, except... $F = m \cdot a = m \cdot \frac{d}{dt} v = m \cdot \frac{d^2}{dt^2} x$

And then there are springs: $F = k \cdot x = k \cdot \int v \, dt = k \cdot \int \int a \, dt \, dt$

Isn't there some way that we could possibly replace all this differentiation and integration with multiplication and division?

For linear systems, where the signals and systems can be represented by Laplace transforms (you'll learn about Laplace transforms in differential equations):

$\frac{d}{dt}$ operation can be replaced with s, and $\int \blacksquare \, dt$ can be replaced by $\frac{1}{s}$

$$\mathbf{X}_{in}(s) \longrightarrow \boxed{\mathbf{H}(s)} \longrightarrow \mathbf{X}_{out}(s) = \mathbf{X}_{in}(s) \cdot \mathbf{H}(s)$$

Transfer function: $\mathbf{H}(s) = \frac{\mathbf{X}_{out}(s)}{\mathbf{X}_{in}(s)}$

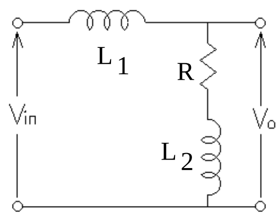
$\mathbf{X}_{in}$ and $\mathbf{X}_{out}$ could be anything from small electrical signals to powerful mechanical motions or forces.

The variable "s" comes from Laplace transformations and is a "complex frequency" and we have moved from the time domain into the "frequency domain". You'll use a version of this extensively in the next class to represent steady-state sinusoidal voltages and currents. In that version s is replaced with $j\omega$. ω is the radian frequency that you've already seen and $j = i = \sqrt{-1}$ We use j rather than i because i is already used for current.

$s = \alpha + j\omega$ and is a complex number, hence the term "complex frequency".

Then our nice, linear, blocks could contain Laplace transfer functions, like this:

Consider a circuit:



$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_{in}(s)} = \frac{R + L_2 \cdot s}{R + L_1 \cdot s + L_2 \cdot s} = \frac{R + L_2 \cdot s}{R + (L_1 + L_2) \cdot s}$$

$$= \frac{L_2 \cdot s + R}{(L_1 + L_2) \cdot s + R}$$

This could now be represented in as a block operator:

$$\mathbf{V}_{in}(s) \longrightarrow \boxed{\frac{L_2 \cdot s + R}{(L_1 + L_2) \cdot s + R}} \longrightarrow \mathbf{V}_o(s) = \mathbf{V}_{in}(s) \cdot \mathbf{H}(s)$$

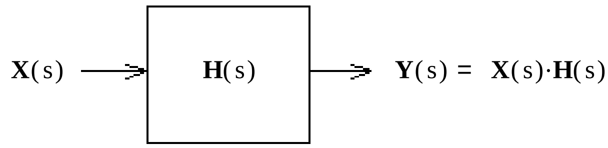
Transfer functions can be written for all kinds of devices and systems, not just electric circuits and the input and output do not have to be similar. For instance, the potentiometers used to measure angular position in the servo have an angle as input and a voltage as output.

Laplace transforms will be important!! Don't let your Differential equations professor put them off until the last week of class.

BUT, that's beyond the scope of this class... So back to just blocks

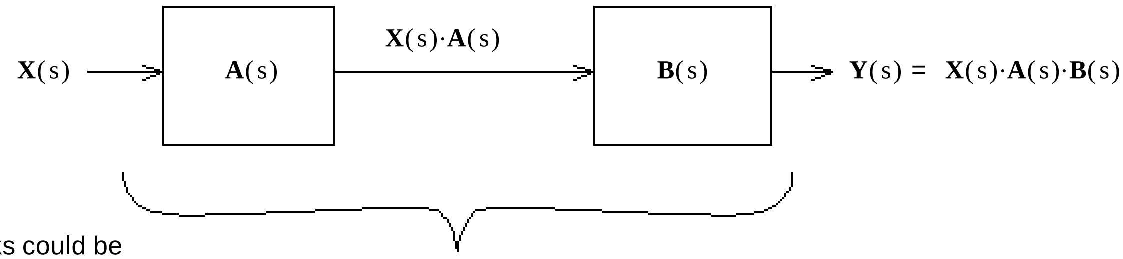
In general:

$$\mathbf{H}(s) = \frac{\text{output}}{\text{input}} = \frac{\mathbf{Y}(s)}{\mathbf{X}(s)}$$

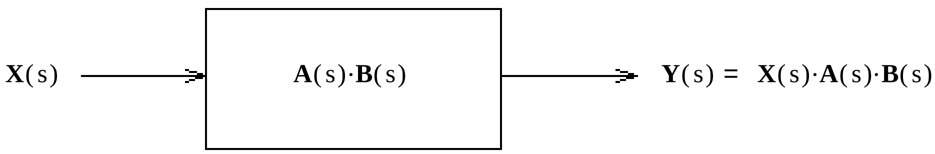


Serial - path systems

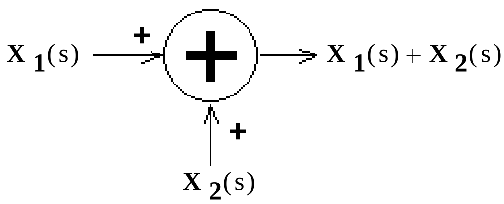
Two blocks with transfer functions $\mathbf{A}(s)$ and $\mathbf{B}(s)$ in a row would look like this:



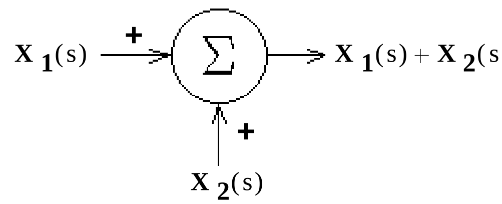
The two blocks could be replaced by a single equivalent block:



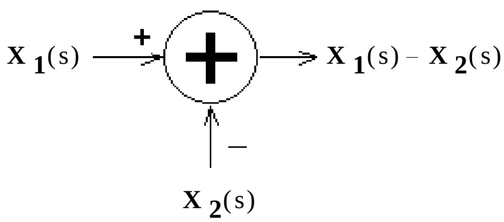
Summer blocks can be used to add signals:



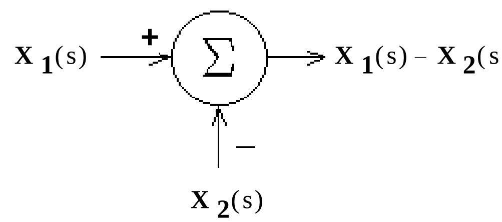
OR



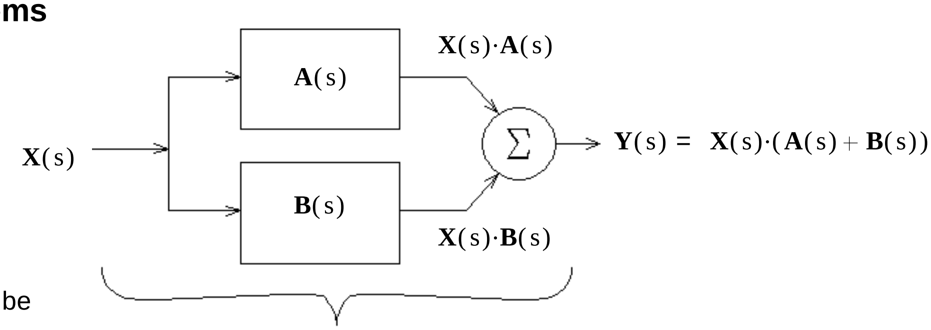
or subtract signals:



OR

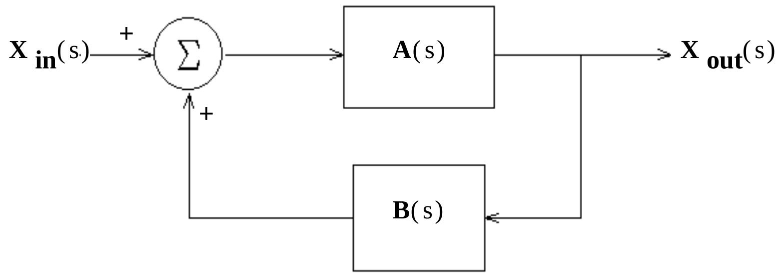


Parallel - path systems



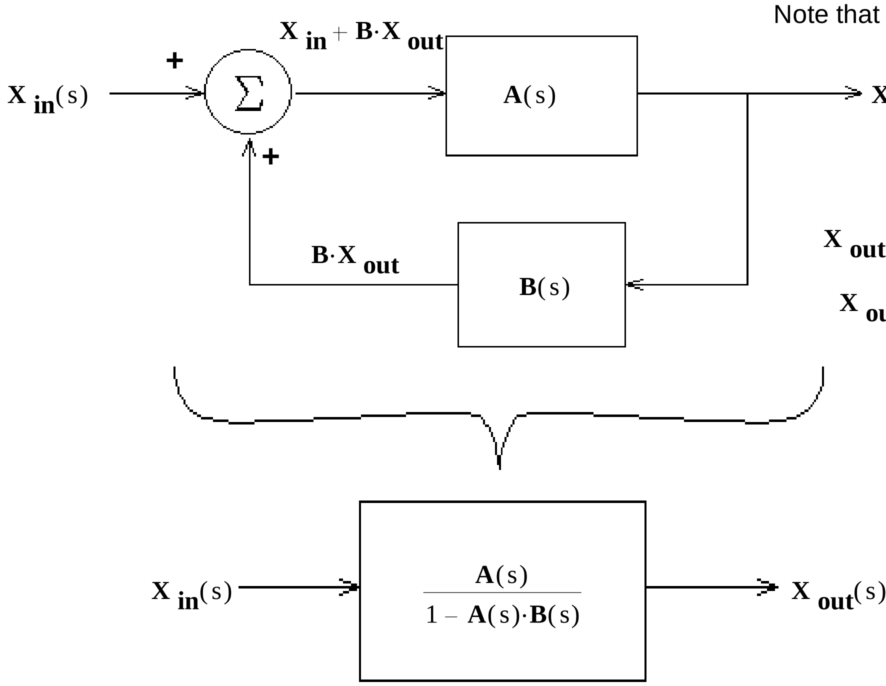
The two blocks could be replaced by a single equivalent block:

A feedback loop system is particularly interesting and useful:



The entire loop can be replaced by a single equivalent block:

Note that I've begun to drop the (s)



$$X_{out}(s) = A \cdot (X_{in} + B \cdot X_{out})$$

$$= A \cdot X_{in} + A \cdot B \cdot X_{out}$$

$$X_{out} - A \cdot B \cdot X_{out} = A \cdot X_{in}$$

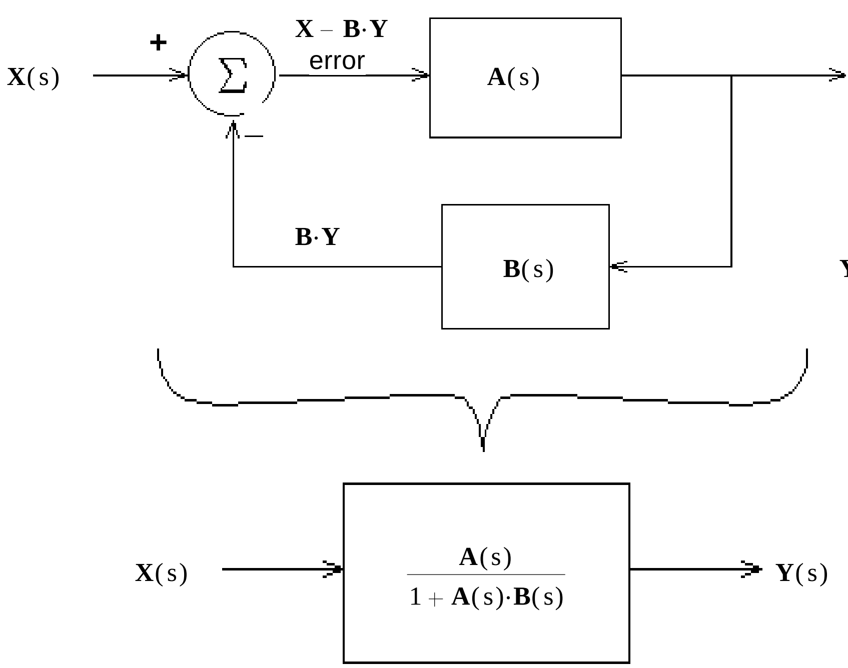
$$X_{out} \cdot (1 - A \cdot B) = A \cdot X_{in}$$

$$\frac{X_{out}}{X_{in}} = \frac{A}{1 - A \cdot B} = H(s)$$

The equivalent transfer function

$A(s) \cdot B(s)$ is called the "loop gain" or "open loop gain"

Negative feedback is more common and is used as a control system:



$$Y(s) = A \cdot (X - B \cdot Y)$$

$$= A \cdot X - A \cdot B \cdot Y$$

$$Y + A \cdot B \cdot Y = A \cdot X$$

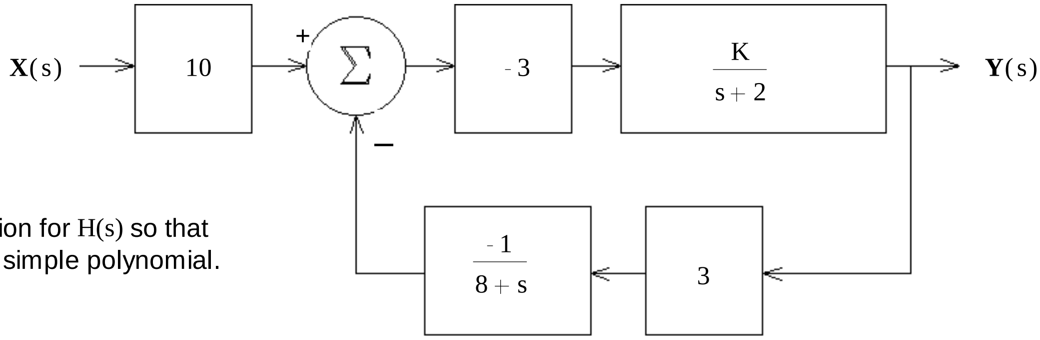
$$Y \cdot (1 + A \cdot B) = A \cdot X$$

$$\frac{Y}{X} = \frac{A}{1 + A \cdot B} = H(s)$$

The equivalent transfer function

This is called a "closed loop" system, whereas a a system without feedback is called "open loop".
The term "open loop" is often used to describe a system that is out of control.

Ex: A feedback system is shown in the figure. What is the transfer function of the whole system, with feedback.



$$\mathbf{H}(s) = \frac{\mathbf{Y}(s)}{\mathbf{X}(s)} = ?$$

Simplify your expression for $H(s)$ so that the denominator is a simple polynomial.

Feedback loop:

Loop gain: $L = \left(\frac{-3\cdot K}{s+2}\right)\cdot\left(\frac{-1\cdot 3}{8+s}\right)$

Simplification:

$$A_f = \frac{\left(\frac{-3\cdot K}{s+2}\right)}{1+\left(\frac{-3\cdot K}{s+2}\right)\cdot\left(\frac{-3}{8+s}\right)}$$

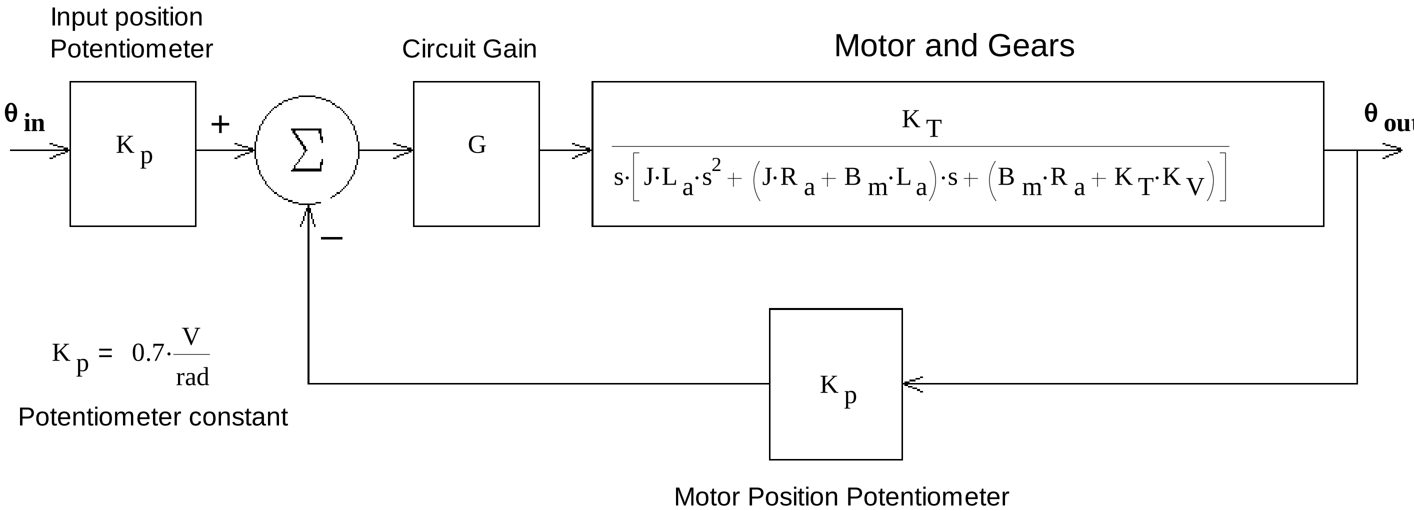
$$A_f = \frac{\left(\frac{-3\cdot K}{s+2}\right)}{1+\left(\frac{-3\cdot K}{s+2}\right)\cdot\left(\frac{-3}{8+s}\right)}\cdot\left[\frac{(s+2)\cdot(8+s)}{(s+2)\cdot(8+s)}\right] = \frac{(-3\cdot K)\cdot(s+8)}{(s+2)\cdot(8+s)+(3\cdot K)\cdot 3}$$

$$= \frac{(-3\cdot K)\cdot s - K\cdot 24}{s^2+10\cdot s+16+9\cdot K}$$

Whole system:

$$H(s) = 10\cdot\frac{-3\cdot K\cdot s - 24\cdot K}{s^2+10\cdot s+16+9\cdot K} = \frac{-30\cdot K\cdot(s+8)}{s^2+10\cdot s+16+9\cdot K}$$

A simple servo system can be represented by:



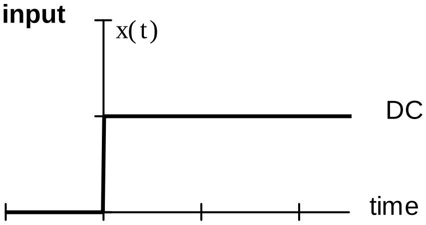
$K_p = 0.7\cdot\frac{V}{rad}$

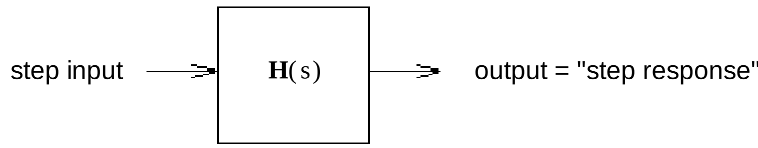
Potentiometer constant

$$\mathbf{H}(s) = \frac{\boldsymbol{\theta}_{\mathbf{out}}(s)}{\boldsymbol{\theta}_{\mathbf{in}}(s)} = \frac{G\cdot K_T\cdot K_p}{s\cdot\left[J\cdot L_a\cdot s^2+\left(J\cdot R_a+B_m\cdot L_a\right)\cdot s+\left(B_m\cdot R_a+K_T\cdot K_V\right)\right]+K_p\cdot G\cdot K_T}$$

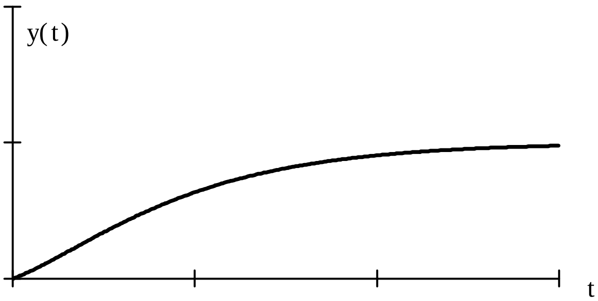
The step response of a system is the output when the input is a step (DC which starts at time-zero).

Step input

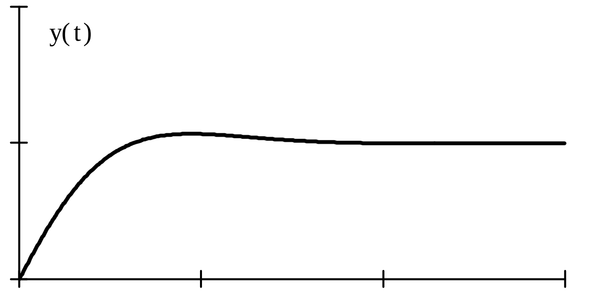




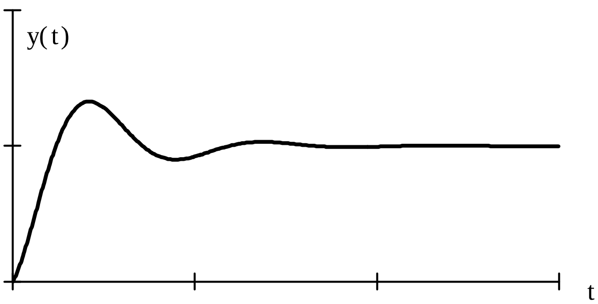
System Step Response



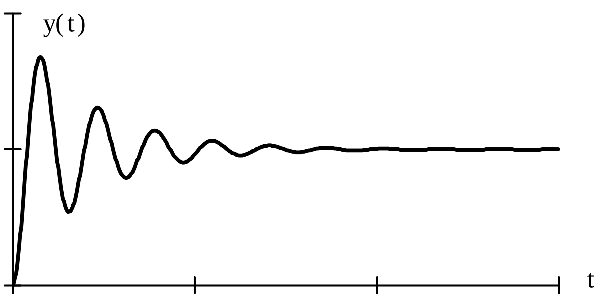
Low gain, response is slow



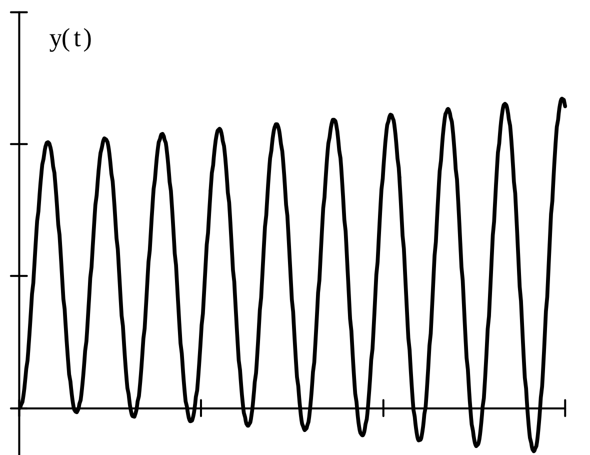
Higher gain, response is faster, but may overshoot



Still higher gain



And higher



Gain is too high

The analysis and design of feedback systems and control systems is the subject of many classes in both Electrical and Mechanical engineering.