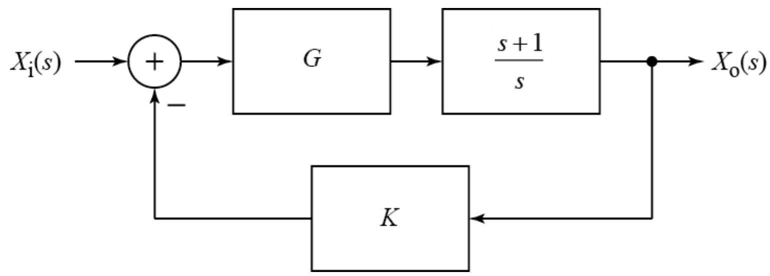
Ex:



a) Find the transfer function, $H(s) = \dfrac{X_o(s)}{X_i(s)}$, for the above system.

b) If $G = 10$, for what values of K is the system stable? (Consider positive and negative values of K.)

SOL'N: a) We multiply the input to a box by the quantity in the box to get the output of the box. We can combine boxes in series by multiplying the quantities they contain. This reduces the above system to one with a single forward-path box containing $G\dfrac{s+1}{s}$. For a forward path of $G\dfrac{s+1}{s}$ and feedback of K, we employ the formula that the transfer function is given by the forward path divided by one plus the product of the forward path and feedback path.

$$H(s) = \frac{X_o(s)}{X_i(s)} = \frac{G\dfrac{s+1}{s}}{1 + G\dfrac{s+1}{s}K}$$

We simplify the expression by multiplying top and bottom by the denominator of the denominator, namely, s.

$$H(s) = \frac{G\dfrac{s+1}{s}}{1 + G\dfrac{s+1}{s}K} \cdot \frac{s}{s} = \frac{G(s+1)}{s + G(s+1)K}$$

Next, we write $H(s)$ in standard form as a constant times a ratio of polynomials in s with the coefficient of the highest power of s being unity.

$$H(s) = \frac{G(s+1)}{s + GKs + GK} = \frac{G(s+1)}{s(1+GK) + GK}$$

or

$$H(s) = \frac{G}{1+GK} \cdot \frac{s+1}{s + \frac{GK}{1+GK}}$$

b) The system is stable when the real parts of the roots of the denominator are all negative. (This corresponds to time-domain solutions of the form e^{st} that will decay over time. If the real part of s is positive, we get a growing exponential.)

To find the roots, we set the polynomial in the denominator equal to zero. Note that the constant term $\frac{G}{1+GK}$ does not change the value of the roots. Here, we have only one root since our denominator is first-order.

$$s + \frac{GK}{1+GK} = 0$$

or

$$s = -\frac{GK}{1+GK}$$

Since the root is real, we just need a positive value of $\frac{GK}{1+GK}$. Since G is positive, we have the following problem:

$$\frac{K}{1+GK} > 0$$

If K is positive, the inequality is satisfied, so we need only consider the case of $K < 0$. To get a violation of the inequality, we would have to have a negative K and a positive $1 + GK$. So we must satisfy the following inequality when $K < 0$.

$$1 + GK < 0$$

or

$$GK < -1$$

or

$$K < -\frac{1}{G}$$

So we must satisfy the above inequality or have positive K:

$$K < -\frac{1}{G} \quad \text{or} \quad K > 0$$