**Ex:**
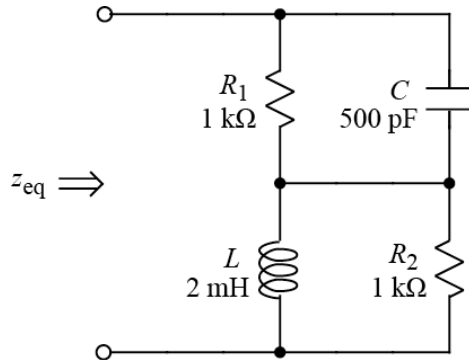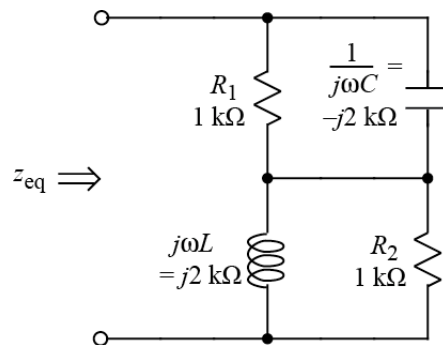


Find the numerical value of the equivalent impedance, $z_{eq}$, for the circuit. Frequency $\omega = 1$ Mr/s. Express your answer in both rectangular and polar form.

**Sol'n:** We convert to the frequency domain by computing impedance values.

$$j\omega L = j(1\text{M})(2\text{m})\Omega = j2\,\text{k}\Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j(1\text{M})(500\text{p})}\Omega = -\frac{j}{500\,\mu} = -j2\,\text{k}\Omega$$

Our frequency domain circuit:



We treat impedance just as we would resistance. We have $R_1$ in parallel with $C$ and $L$ in parallel with $R_2$.

$$z_{eq} = 1\text{k}\Omega \,\|\, -j2\,\text{k}\Omega + j2\,\text{k}\Omega \,\|\, 1\text{k}\Omega = 1\text{k}\Omega \cdot (1 \,\|\, -j2 + j2 \,\|\, 1)$$

or

$$z_{eq} = 1\text{k}\Omega\left(\frac{1(-j2)}{1-j2} + \frac{j2(1)}{j2+1}\right) = 1\text{k}\Omega\left(\frac{-j2}{1-j2} + \frac{j2}{1+j2}\right)$$

We can use a common denominator and rationalize at the same time, since the denominators are complex conjugates.

$$z_{eq} = 1\,k\Omega \left( \frac{-j2}{1-j2} \cdot \frac{1+j2}{1+j2} + \frac{j2}{1+j2} \cdot \frac{1-j2}{1-j2} \right)$$

or

$$z_{eq} = 1\,k\Omega \left( \frac{4-j2}{1^2+2^2} + \frac{4+j2}{1^2+2^2} \right) = 1\,k\Omega \left( \frac{4-j2+4+j2}{5} \right)$$

or

$$z_{eq} = 1\,k\Omega \left( \frac{8}{5} \right) = 1.6\,k\Omega$$

The answer turns out to be real because the two impedances in series are complex conjugates of each other.

Expressing our value in rectangular (first) and polar form (second), we have the following expressions. (Note that writing $z_{eq} = 1.6$ k$\Omega$ is, in a sense, both a rectangular and polar form and is perfectly acceptable.)

$$z_{eq} = 1.6\,k\Omega + j0\,k\Omega \quad \text{or} \quad z_{eq} = 1.6\,k\Omega\angle 0°$$

Note that the units are often placed only at the end of the value rather than inside. The author prefers the above notation that keeps units closer to numerical quantities.